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# THE PHYSICAL REVIEW

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PHYSICAL REVIEW

# HIGH FREQUENCY RAYS OF COSMIC ORIGIN I. SOUNDING BALLOON OBSERVATIONS AT EXTREME ALTITUDES

By R. A. Millikan and I. S. Bowen

Abstract

**Discharge rate of an electroscope at altitudes from 5 to 15.5 km.**—Four specially designed instruments, each comprising a recording electroscope, thermometer and barometer and each weighing but 190 gr were sent up with sounding balloons from Kelly Field, Texas. Three were recovered and of these two had satisfactory records of their flight during which they reached altitudes of 11.2 and 15.5 km, respectively. A comparison of the recorded electroscope reading at the 5 km level during ascent with the reading at the same level during descent shows that the average discharge rate of the electroscopes while above the 5 km level was about three times their discharge rates at the surface of the earth, and corresponded to an average rate of production of ions of 46.2 ions per cc per sec. This is only 25 percent of the value to be expected from the observations of Hess and of Kolhörster and constitutes definite proof that there exists no penetrating radiation of cosmic origin having an absorption coefficient as large as 0.57 per meter of water.

IT was as early as 1903 that the Canadian physicists McLennan and Burton[1] and Rutherford and Cooke[2] noticed that the rate of leakage of an electric charge from an electroscope within an air-tight metal chamber could be reduced as much as 30 percent by enclosing the chamber within a completely encircling shield or box with walls several centimeters thick. This meant that the loss of charge of the enclosed electroscope was not due to causes inside the electroscope but must rather be due to some highly penetrating rays, like the gamma rays of radium, which could pass through metal walls as much as a centimeter thick and ionize the gas inside.

In view of this property of passing through relatively thick metal walls in measurable quantity, the radiation thus investigated was called the "penetrating radiation" of the atmosphere, and was at first quite naturally attributed to radioactive materials in the earth or air, and this is in fact the origin of the greater part of it. But in 1910 and 1911 it was found

[1] McLennan and Burton, Phys. Rev. **16,** p. 184 (1903).
[2] Rutherford and Cooke, Phys. Rev. **16,** 183 (1903).

that it did not decrease as rapidly with altitude as it should upon this hypothesis. The first significant report upon this point was made by the Swiss physicist, Gockel,[3] who took an enclosed electroscope up in a balloon with him to a height of 4500 meters and reported that he found the "penetrating radiation" about as large at this altitude as at the earth's surface, and this despite the fact that according to Eve's[4] calculation it ought to have fallen to half its surface value in going up 250 feet.

In 1911, 1912, 1913, and 1914 two other European physicists, Hess[5] an Austrian, and Kolhörster,[6] a German, repeated these balloon-measurements of Gockel's, the latter going up to a height of 9 km, or 5.6 miles, and reported that they found this radiation decreasing a trifle for the first kilometer or so and then increasing until it reached a value at 9 km, according to Kolhörster's measurements, seven times as great as at the surface. This seemed to indicate that the penetrating rays came from outside the earth, and were therefore of some sort of cosmic origin. If so it was computed[7] that in order to fit the Hess and Kolhörster data, the rays had to have an absorption coefficient of .57 per meter of water and an ionizing power within a closed vessel sent to the top of our atmosphere of at least 500 ions per cc per sec., in place of the 10 or 12 ions found in ordinary electroscopes at the surface.

The following table gives a summary of all the Hess-Kolhörster data which in 1914 stimulated the interest of one of us in the problem and furnished the basis for the computations with the aid of which our sounding-balloon apparatus was designed. The earliest of these designs was made in 1915–16, but was not completed because the war put a stop

TABLE I

*Hess*

| Altitude in Meters | Ions per cc per sec. in excess of ground value Instrument No. 1 | Instrument No. 2 |
|---|---|---|
| 0 – 200 | − 0.9 | − 0.7 |
| 200 – 500 | − 0.8 | − 1.4 |
| 500 – 1000 | − 0.7 | − 1.5 |
| 1000 – 2000 | − 0.4 | + 0.3 |
| 2000 – 3000 | + 1.0 | + 1.5 |
| 3000 – 4000 | + 3.5 | + 4.7 |
| 4000 – 5200 | +18.1 | +15.4 |

[3] Gockel, Phys. Zeits. **11**, 280 (1910).

[4] Eve, Phil. Mag. **21**, 26 (1911).

[5] Hess, Phys. Zeits. **12**, 998 (1911), and **13**, 1084 (1912).

[6] Kolhörster, Phys. Zeits. **14**, 1153 (1913), and Verh. d. Deut. Phys. Ges. (July 30, 1914).

[7] H. von Schweidler, Elster u Geitel Festschrift, p. 411, 1915.

*Kolhörster*

| Flight 1 | | Flight 2 | | Flight 3 | | Flight 4 | |
|---|---|---|---|---|---|---|---|
| Alt. | Ions | Alt. | Ions | Alt. | Ions | Alt. | Ions |
| 310 | − 1.2 | 500 | − 2.0 | 1090 | − 1.2 | 1000 | − 1.5 |
| 760 | − 1.3 | 600 | − 1.4 | 2130 | + 2.1 | 2000 | + 1.2 |
| 1650 | + 0.8 | 1000 | − 2.1 | 3550 | + 7.0 | 3000 | + 4.3 |
| 2110 | + 1.3 | 1400 | − 1.7 | 4700 | +14.5 | 4000 | + 9.3 |
| 2400 | + 3.1 | 1500 | − 0.8 | 5600 | +27.5 | 5000 | +17.2 |
| 2600 | + 4.3 | 2400 | + 3.1 | 6200 | +29.3 | 6000 | +28.7 |
| 3000 | + 7.5 | 3300 | + 4.5 | | | 7000 | +44.2 |
| 3400 | + 8.9 | 4000 | + 6.7 | | | 8000 | +61.3 |
| 3500 | +11.1 | | | | | 9000 | +80.4 |

to further activity in this direction. Upon the cessation of the war about two years were spent in trying to have built the right kind of sounding-balloons. In the winter of 1921–22 with the aid of Mr. Julius Pearson we designed and had constructed in the Norman Bridge Laboratory of Physics four little recording electroscopes. These we took to Kelly Field near San Antonio, Texas, in the spring of 1922, for the purpose of attempting to get them as near as possible to the top of the atmosphere in order to obtain a crucial test as to whether there is such a cosmic radiation as the Hess-Kolhörster data seemed to require.

These electroscopes were made of steel 0.3 mm thick, and were designed to support an internal pressure of 10 atmospheres, our intention being to increase, if necessary, the rate of leak by using air at high pressure. Actually, however, the two successful flights were made at atmospheric pressure, since Kolhörster's data indicated that there existed a penetrating radiation of such intensity as completely to discharge the electroscope during the flight if the ten atmospheres pressure were used. In addition all the electroscopes carried photographic films and the necessary driving mechanism for obtaining a continuous record during the ascent and descent (1) of the divergence of the electroscope fibres, (2) of the temperature and (3) of the barometric pressure. The total weight of each with all its contents was but 190 grams, or about 7 ounces.

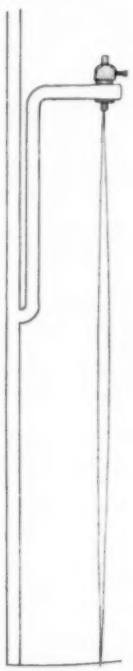

Fig. 1.

Plate 1a shows a photograph of one of these instruments ready for flight, and placed beside it a six-inch (15 cm) rule. Plate 1b shows the inner mechanism. The electroscope consists of two sputtered quartz fibres such as are used in a Wulf electroscope, the deflection of which was produced by charging with a 300 volt battery through the movable charging-rod *C*. To make them as free from temperature effects as possible, all the supports were of quartz and of the shape shown in Fig. 1.

The lower spring was a single quartz fibre, having suitable elastic properties, rigidly attached at one end only. These supports are all inside the metal shield $M$, Plate 1b, provided to eliminate the effect of static charges on the quartz. A shadow image (diffraction pattern) of the fibres produced by the light from the sky passing through an exceedingly fine vertical slit $S_1$ carried by the slit holder shown at the left of the rule in Plate 1a fell upon the photographic film through a wide horizontal slit $S_2$ on the opposite side of the cylinder. The thermometer $T$ was a small coil of duo-metal carrying a minute pointer $P$ which moved up and down a

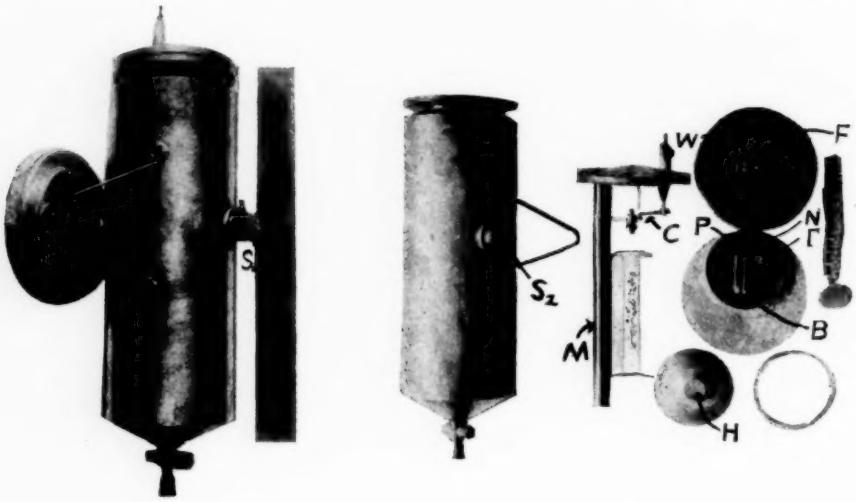


Plate 1. a. The complete recording instrument ready for flight. A 6-inch (15 cm) rule is shown beside it for comparison.

b. The inner mechanism.

vertical slit $N$ behind which the photographic film was moving, the whole being illuminated by skylight entering through a fine horizontal slit $H$ in the cap which fitted, light tight, over the thermometer and barometer chamber. The barometer $B$ is a small, closed-arm manometer, the top of the liquid surface in one arm of which is also registered on the photographic film by skylight entering through $H$. The two circular films and the black paper separating them were fastened on the carrier $F$ which was directly connected to the main spring of the small watch $W$.

Each of these electroscopes was carried up by two balloons eighteen inches across when deflated and weighing about 300 grams apiece. These balloons were specially made from our specifications by the Sterling

Rubber Company of Guelph, Canada. They were inflated with hydrogen to a buoyancy of 550 grams each (diameter about 95 cm) and rose at a rate of 130 meters per minute when carrying between them a total load of 220 grams (electroscope, cord, directions for return and so forth).

The rates of ascent were measured by the two-theodolite method, Lieutenant McNeal was especially sent from Washington to Kelly Field by Major William R. Blair of the U. S. Signal Corps to assist us in these experiments by making these two-theodolite altitude measurements.

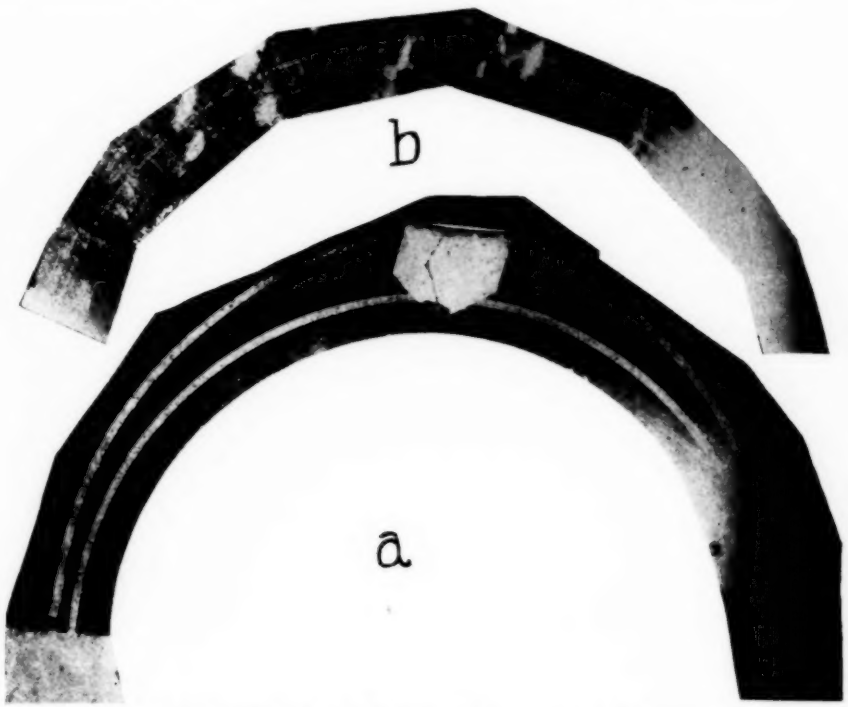


Plate 2. a. Reproduction of the photographic film giving the temperature record of the 15.5 km flight.

b. Reproduction of the photographic film giving the divergence of the electroscope fibers during the 15.5 km flight.

The purpose of sending up two balloons with each instrument is not merely to gain additional buoyancy but to enable one of them to bring the instrument safely to earth after the other has burst and also to serve, after the return to earth, to attract attention and increase the likelihood of the return of the instrument to the sender as per instructions attached to it.

We succeeded in making four flights and had three instruments returned, two of which had reached the altitudes of 11.2 and 15.5 kilo-

meters respectively. They were returned from distances about eighty miles away from the starting point.

Plate 2a is a reproduction of the photographic film giving the temperature record of the 15.5 kilometer flight. The film was driven at the rate of 48° to the hour, the total time of flight as shown by this film was three hours and eleven minutes. This was accurately determined by plotting as in Fig. 2 the slopes of the time-temperature curve going up and coming down. The point of intersection of these two slopes gives the exact time at which one of the balloons burst and the descent began. The temperature at this time was in this case $-60°$C, as measured by the divergence

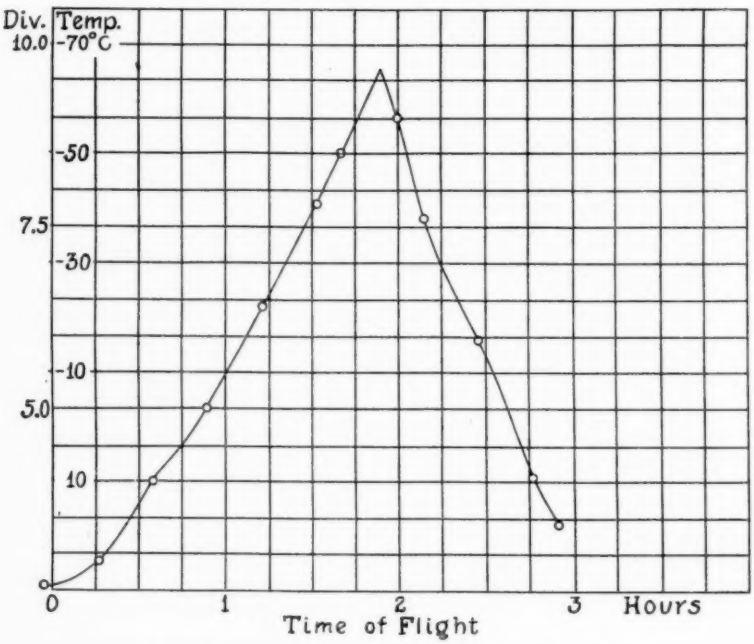


Fig. 2. The time-temperature curve for one flight.

of the two traces, a divergence read with a microscope and accurately calibrated by preceding laboratory tests down to $-77°$C. The horizontal edge on the left side of the blackened portion of the film gives the instant at which the driving mechanism was started just before the beginning of the flight. In this particular flight the apparatus reached the earth just after sunset and for this reason the thermometer record vanishes before the temperature has quite returned to the initial value, but the rate of descent is altogether uniform so that no uncertainty is thereby introduced into the time of flight. The time of ascent was in this case 115 minutes and that of descent 76, the total being 191 minutes. The

relative times of ascent and descent are determined accurately from the ratio of the slopes of the ascending and descending portions of the temperature curve.

Plate 2b is a reproduction of the photographic record of the divergence of the electroscope fibres during the three hours and eleven minutes of this flight. It will be seen that the two lines representing the shadows of the fibres on the film lend themselves very well to accurate microscopic measurement. Indeed, there was a very sharp diffraction edge on each side of each line which made the setting particularly exact. On account of the fact that we were interested only in discharge rates at high altitudes we used for our computation of these discharge rates the initial measurements of the deflection of the fibres when the instrument had reached an altitude of 5 kilometers on the ascent, and final measurements of the fibre-deflection when it had reached again the same altitude of 5 kilometers on the descent. Since the temperature of the instrument is the same at a given altitude while going up as while coming down, our discharge rates are thus made entirely independent of the influence of temperature upon the elastic properties of the fibres and supports—a matter that we have found to be of greatest importance.

Fig. 3 contains the graphical representation of all of the electroscope readings as well as the time-altitude determinations, abscissas being time in hours and the scale of ordinates to the right, altitude in kilometers, while the scale of ordinates to the left represents fibre-deflections. The straight sloping lines meeting at the top represent the constant rates of ascent and of descent, the first of which are obtained from direct theodolite measurements.

The line *G* at the top sloping gently downward is the rate of discharge in scale divisions while the instrument was at the surface. This discharge rate is very accurate since it was taken over a discharge period of twenty-four hours. It will be seen that the straight line *F* connecting the two groups of points representing measurements at the two 5 kilometer altitudes has a slope corresponding to a mean discharge rate three times that found at the earth's surface. *This shows quite unambiguously, in agreement with the findings of Gockel, Hess, and Kolhörster, that the discharge rates at high altitudes are larger than those found at the surface.* Quantitatively, however, there is complete disagreement between the Hess-Kolhörster data and our own, the total loss of charge of our electroscope in the two hours spent between the altitudes of 5 km and 15.5 km having been but about 25 percent of that computed from the Hess-Kolhörster curve which is represented by the curved line *K* of Fig. 3. The data plotted in Fig. 3 also shows very beautifully that, despite the

extraordinary precautions herein taken to make an electroscope whose readings would be independent of temperature, there was nevertheless a marked temperature effect. Thus Fig. 3 shows that the apparent deflection reaches a minimum when the temperature is a minimum and then rises again as the instrument warms up in the descent. *Our results are entirely free from these temperature effects since our initial and final readings are taken under the same temperature conditions.*

The mean rate of discharge in the ascent from 5 km to 15.5 km and the descent again to 5 km of this electroscope was 46.2 ions per cc per sec. The average rate of discharge of the same electroscope at the surface

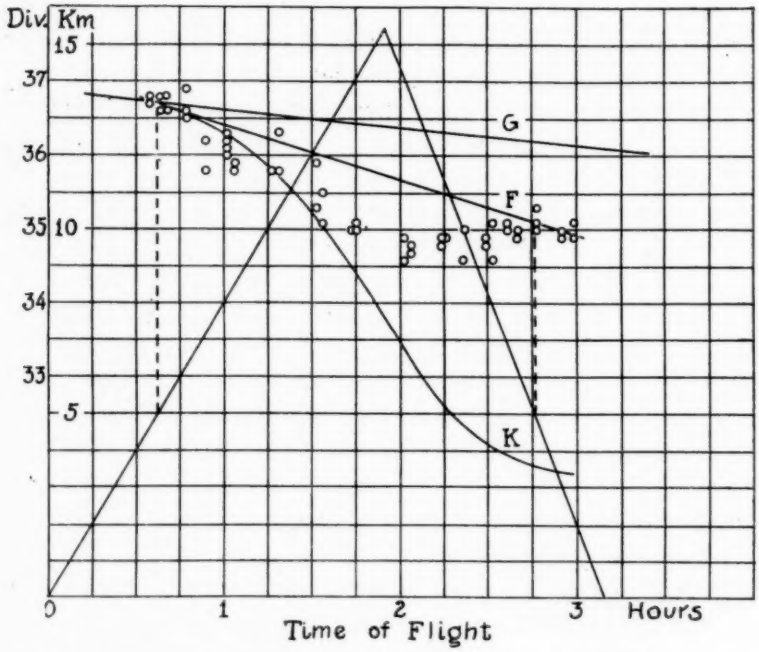


Fig. 3. Electroscope readings and altitude determinations as functions of the time.

was 15.4 ions per cc per sec., so that the mean rate of discharge at these very high altitudes here attained was but three times the surface rate, in contrast with the sevenfold rate shown by the data in Table 1 at the lower altitude of 9 km.

Another flight which reached an altitude of 11.4 km also yielded a total loss of charge not more than 25 percent of that found by the European observers, though some temperature changes rendered its evidence much less reliable than that of the 15.5 km flight.

The results then of the whole Kelly Field work constitute definite proof that there exists no radiation of cosmic origin having such characteristics

as we had assumed. They show that the ionization increased much less rapidly with altitude than would be the case if it were due to rays from outside the earth having an absorption coefficient of .57 per meter of water.

Taken in conjunction, however, with experiments on absorption coefficients to be reported in the succeeding articles II and III—experiments which present unambiguous evidence for the existence of a cosmic radiation of extraordinary penetrating power, $\mu$ calculated as above being as low as .18 per meter of water, these experiments at very high altitudes have important bearings upon the distribution in wave-length of these hard rays as they enter the atmosphere.

NORMAN BRIDGE LABORATORY OF PHYSICS,
CALIFORNIA INSTITUTE OF TECHNOLOGY,
PASADENA, CALIFORNIA,
December 24, 1925.

# A CORRESPONDENCE PRINCIPLE IN THE COMPTON EFFECT*

By G. Breit

Abstract

**Correspondence theorem for frequencies.**—It is well known that the frequency emitted by a hydrogen atom as it falls from one of its quantized states to another may be expressed as a mean value of the frequency of motion of the electron (or overtone thereof) when averaged in the proper manner over orbits intermediate between the initial and final states. In the present paper it is *proved* that, similarly, *the Compton shift is a properly taken mean of the classical Doppler shift.* The quantum frequency actually scattered is thus a properly taken average of the frequency which would be scattered on the classical theory as the electron is accelerated from its state of rest to its final recoil condition.

**Correspondence principle for intensities.**—In like manner, the amount of light scattered in various directions may be determined if it is *assumed* that the intensity in the quantum theory equals a proper average of the intensities scattered according to the classical theory. A comparison is made with observed data on the scattering of $\gamma$-rays.

The characteristic feature of the present paper is that *the corresponding classical electron is assumed to have the same direction of motion as the scattered quantum,* whereas an actual classical electron would from symmetry recoil straight forward in the direction of the incident beam, as in Compton's and Woo's theories of intensities. This new point of view eliminates the difficulty of a constant correction-factor which has been encountered by Compton and Woo in their explanation of intensity relations.

## 1. Introduction

THEORIES giving the dependence of the intensity of scattered radiation upon the angle have been given by Compton, Debye, and Woo. The theory of Debye is in poor agreement with experiment. The theories of Compton and Woo agree with experiment except for the occurrence of a factor $1+2\Lambda/\lambda_0$ which must be omitted in order to satisfy the experimental facts. The theory of Compton and Woo postulates that the intensity of the scattered radiation may be obtained by considering the classical scattering by an electron having the velocity $c\Lambda/(\Lambda+\lambda_0)$, where $c$ is the velocity of light, $\lambda_0$ the wave-length of the incident radiation, and $\Lambda=h/m_0c$, $h$ being Planck's constant and $m_0$ the rest mass of the electron. The reason for this assumption as given by Compton is that such an electron scatters radiation of the right frequency and that, therefore, it may also be expected to scatter it with the right intensity.

* Presented at the Kansas City meeting of the American Physical Society, Dec. 1925.

*A priori* there is no objection to this reasoning. However, the calculations of the above authors show that their picture is not altogether satisfactory. The factor by which their result must be divided is just such as to give the classical intensity in the direction of the incident radiation. This fact in itself suggests another point of view.

The emitting atom and the scattering electron may be conceived as one complex atom. The scattering process is the emission process of that atom. In its initial state the electron is at rest, and in the final state the electron is in motion. According to Bohr's correspondence principle we should expect that both *the frequency of the emitted radiation and its intensity are given by proper averages of the corresponding quantities between the initial and final states*. It is clear without calculation that such a point of view leads us to expect the same intensity in the direction of incident radiation as is demanded by the classical theory, because in this case there is no recoil of the electron. We first show that the frequency actually scattered is a properly taken average of the frequency scattered classically in the initial and final states of the scattering electron. We then calculate the intensity of scattered radiation in the initial and final states and we show that a proper mean of these values is in agreement with experiment. Even though we apparently adhere in this to the prescriptions of the correspondence principle, we cannot altogether put this problem on the same footing as the application of the correspondence principle to the atomic frequency theorem of Bohr or to the selection rules. To do this it would be necessary to obtain a description of the possible atomic states by means of phase integrals.

## 2. Correspondence Theorem for Frequencies

The quantum theory of the Compton effect is governed by the relations

$$\begin{aligned} &\frac{m_0 v}{\sqrt{1-\beta^2}}\cos\varphi+\frac{h\nu_\theta}{c}\cos\theta=\frac{h\nu_0}{c}\\ &-\frac{m_0 v}{\sqrt{1-\beta^2}}\sin\varphi+\frac{h\nu_\theta}{c}\sin\theta=0 \qquad \text{(I)}\\ &m_0c^2\left(\frac{1}{\sqrt{1-\beta^2}}-1\right)+h\nu_\theta=h\nu_0 \end{aligned}$$

where the electron is supposed to be at rest before scattering. In these relations $h\nu_0$ and $h\nu_\theta$ are the magnitudes of the original and scattered quanta, respectively; $\theta$, the angle between their directions; $\varphi$, the angle between the velocity $v$ of the electron after scattering and the incident

radiation; and $\beta$ is $v/c$. As is known, when these equations are solved for $\nu_\theta$ one finds

$$\nu_\theta = \frac{\nu_0}{1+\frac{\nu_0}{N}(1-\cos\theta)} \tag{II}$$

where

$$N = \frac{m_0 c^2}{h} . \tag{II'}$$

In order to determine the frequency which would be scattered by the electron in its final recoil state we must consider first of all what the incident frequency appears to be to an observer on the moving electron. We must next compute how the scattered radiation is transformed to the stationary frame of reference. The system of the recoil electron we refer to as $K'$ and to that of the stationary one we refer as $K$. In the

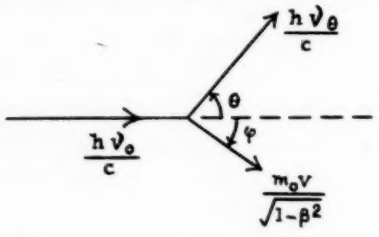


Fig. 1.

equations of the Lorentz transformation we take the $X$ axis along the direction of recoil. The fundamental relation is

$$\frac{\nu}{\nu'} = \frac{1+\beta\cos\psi'}{\sqrt{1-\beta^2}} = \frac{\sqrt{1-\beta^2}}{1-\beta\cos\psi} \tag{III}$$

where $\Psi$ is the angle with the $X$ axis referred to $K$, $\Psi'$ is the same angle referred to $K'$ and $\nu$, $\nu'$ are the frequencies of the radiation referred to $K$, $K'$ respectively. We have, therefore, for the frequency scattered by the recoil electron on the basis of classical relativity

$$\frac{\nu_{\theta c}}{\nu_0} = \frac{1-\beta\cos\varphi}{1-\beta\cos(\theta+\varphi)} \tag{1}$$

It follows from (1) that

$$\beta\cos(\theta+\varphi) = \frac{\nu_0\cos\theta-\nu_\theta}{N+\nu_0-\nu_\theta} \tag{2}$$

$$\beta\cos\varphi = \frac{\nu_0-\nu_\theta\cos\theta}{N+\nu_0-\nu_\theta} \tag{3}$$

Whence using (II) we have

$$\frac{\nu_{\theta c}}{\nu_0}=\frac{N-\nu_\theta(1-\cos\theta)}{N+\nu_0(1-\cos\theta)}=\frac{1}{\left(1+\frac{\nu_0}{N}(1-\cos\theta)\right)^2}=\left(\frac{\nu_\theta}{\nu_0}\right)^2 \tag{4}$$

This means that

$$\log\nu_\theta=\frac{1}{2}[\log\nu_0+\log\nu_{\theta c}] \tag{5}$$

In other words the logarithm of the scattered frequency is the arithmetic mean of the frequency which would be scattered in the initial and in the final states on the basis of the classical theory. If $(\nu_\theta-\nu_0)/\nu_0<<1$ we also have approximately $\nu_\theta=\frac{1}{2}(\nu_0+\nu_{\theta c})$ as may be shown by a simple consideration of the vector diagram.

It is hardly necessary to point out that Eq. (5) is not altogether analogous to Bohr's Correspondence Theorem, for in that theorem attention is paid to the classical frequency in every intermediate step. In (5), however, we only use the mean of the end values of the classical frequency. It is perhaps dangerous to try to use a method entirely analogous to Bohr's because the Compton shift does not exist on the classical theory.

In a purely speculative manner, however, we may try to look at the problem from the following point of view. Since the quantum emission process is entirely unidirectional, the "corresponding" classical picture we take to be also entirely unidirectional. The classical plane wave is incident on the scattering electron and gradually imparts to it its final momentum. At any intermediate stage in this scattering process the electron is executing a recoil motion which, however, is smaller than the final one. We shall attempt to justify presently the assumption that the classically scattered frequency must be averaged by regarding $h$ in (4) as a variable having a range between 0 and $6.55\times10^{-27}$. The weights of equal intervals in this range we consider as equal. Letting

$$a=\frac{1-\cos\theta}{m_0c^2}\,, \tag{6}$$

the average frequency emitted is according to (5)

$$\bar{\nu}_{cl}=\frac{1}{h}\int_0^h\frac{\nu_0dx}{(1+a\nu_0x)^2}=\frac{\nu_0}{1+a\nu_0h}=\nu_\theta\,, \tag{7}$$

as is seen from (II). This means that the scattered frequency is a properly taken average of the classically scattered frequency.

The justification for taking the average in the manner indicated by (7) may be seen in the fact that Bohr's correspondence theorem assigns equal weights to equal elements of length on the straight line in $n$-dimensional space joining $(J_1, J_2, \ldots)$ to $(J_1+n_1h, J_2+n_2h, \ldots)$. Considering for simplicity the one-dimensional case, the frequency emitted is $n(\partial W/\partial J)$ where $W$ is the energy and where the average is taken between $J$ and $J+nh$, equal weights being attached to all elements $dJ$. It is clear that equal weights are attached to all intervals corresponding to equal energy intervals as long as $W$ increases uniformly with $J$. In general $W$ does not increase uniformly with $J$, this being rigorously the case only for a linear oscillator. The averaging assumption included in (7) means that equal values of $\Delta W$ referred to the original emitting atom correspond to equal weights in the scattering mechanism. To this extent we suppose the emitting atom to be equivalent to a linear resonator at a large distance.

We think, therefore, of the emitting atom and the scattering electron as one complex atom. We have shown by means of II, (4), (7) that the frequency $\nu_0$ which is actually scattered is the average of the frequency which would be scattered on the classical theory provided the *energy* extracted from the quantum incident on the electron determines the linear scale for averaging.

### 3. Correspondence Principle for Intensities

We may now pass on to the intensity relations. We need first of all to know what intensity the classical theory would give for the radiation scattered in the final recoil state. We adopt as a criterion the intensity scattered per unit solid angle in the direction $\theta$ of the actually scattered radiation. We shall simplify the calculation by showing that in the frame of reference of the recoil electron the incident and recoil quanta make with each other the same angle $\theta$ as in the frame of reference of the electron before scattering. This fact, as well as others useful in connection with the present problem, can be obtained most simply by using Pauli's very elegant treatment of the interaction between quanta and electrons.[1] Pauli denotes the energy of the quantum by $E$, its momentum by $\Gamma$, the energy of the electron by $U$ and its momentum by $G$. He then shows that if a frame of reference be chosen such that in it $\Gamma$ and $G$ are equal and opposite before impact, then after impact $\Gamma$ and $G$ are equal and opposite,

[1] W. Pauli, Zeits. f. Physik **18**, 272 (1923).

[2] Laue, Die Relativitätstheorie, Vol. I, p. 124, formula 177; Vieweg, 1921.

and, further, the new values of $\Gamma$ and $G$ as well as the new values of $E$ and $U$ are all equal respectively to their old values. The only change in that frame which occurs at collision is a change in the direction of all the vectors without change in absolute magnitude. It is clear from this fact and the symmetry of Fig. 2 that a Lorentz transformation to a frame of reference moving with velocity $v$ along $G$ leads to the same result for the angle between $\Gamma$ and $\Gamma'$ as the Lorentz transformation to a frame moving with the same velocity $v$ along $G'$. This proves that in the frame of the recoiling electron the angle between the incident and the reflected quantum is the same angle $\theta$ as that formed in the frame of the electron before collision.

It further proves that in the frame of the recoiling electron the reflected quantum has the magnitude $h\nu_0$ and the incident quantum has the magnitude $h\nu_\theta$, while in the frame of the electron before collision the reflected

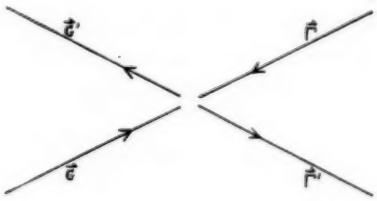


Fig. 2.

quantum has the magnitude $h\nu_\theta$ and the incident one is $h\nu_0$. (The magnitude of the quantum is the same if referred to the electron before and after collision.)

Suppose now in $K'$ (the frame of the recoil electron) the amount of energy radiated into the angular domain $d\theta'$, $d\varphi'$ is

$$f(\theta',\varphi')\sin\theta'\,d\theta'\,d\varphi' \tag{8}$$

then in the stationary frame of reference $K$ this radiation appears to have an energy content greater than the above in the ratio $\nu^2:\nu'^2$ so that an amount of energy

$$\frac{\nu^2}{\nu'^2}f(\theta',\varphi')\sin\theta'\,d\theta'\,d\varphi' \tag{9}$$

is radiated into the corresponding angular domain $d\theta$, $d\varphi$.[2]

Since

$$\sin\theta'\,d\theta'\,d\varphi'=\frac{\nu^2}{\nu'^2}\sin\theta\,d\theta\,d\varphi \tag{10}$$

we have an amount of energy

$$\left(\frac{\nu}{\nu'}\right)^4 f(\theta',\varphi') \tag{11}$$

per unit solid angle in $K$. In this formula it is important to remember that $\nu/\nu'$ refers to one particular direction of radiation, namely, that of the recoiling quantum. Therefore, here we must write

$$\frac{\nu}{\nu'} = \frac{\nu_\theta}{\nu_0} \tag{12}$$

making the energy distribution

$$\left(\frac{\nu_\theta}{\nu_0}\right)^4 f(\theta',\varphi') \tag{13}$$

The expression for $f(\theta', \varphi')$ is obtained by considering what becomes of the incident radiation when it is transformed to the frame of the recoil electron. If the intensity of this radiation in $K$ is $I_0$ and if in $K'$ it is $I_0'$ the value of $f(\theta', \varphi')$ is given by the Thomson expression

$$f(\theta',\phi') = CI_0'\frac{1+\cos^2\theta}{2} \tag{14}$$

where

$$C=\frac{e^4}{c^4m^2}$$

because the angle between the scattered and incident radiation we showed to be $\theta$ in $K'$. Further

$$I_0' = \left(\frac{\hat{\nu}'}{\hat{\nu}}\right)^2 I_0 \tag{15}$$

where $\hat{\nu}'/\hat{\nu}$ is the quotient of the frequencies computed by the Lorentz transformation for a ray in the direction of the incident quantum. Since in $K$ this quantum is $h\nu_0$ and in $K'$ it is $h\nu_\theta$

$$\frac{\hat{\nu}'}{\hat{\nu}} = \frac{\nu_\theta}{\nu_0} \tag{16}$$

and the amount of energy scattered per unit time by the recoil electron from a beam of radiation of intensity $I_0$ is

$$\mathcal{I}_\theta = CI_0\left(\frac{\nu_\theta}{\nu_0}\right)^6\frac{1+\cos^2\theta}{2} = \frac{CI_0}{\left[1+\frac{\nu_0}{N}(1-\cos\theta)\right]^6}\,\frac{1+\cos^2\theta}{2} \tag{17}$$

where use has been made of II. We write this also as

$$\mathcal{J}_\theta = I_{\theta c}\left[1+\frac{\nu_0}{N}(1-\cos\theta)\right]^{-6} \tag{18}$$

where $I_{\theta c}$ is the classical expression of J. J. Thomson, which does not take into account the Compton shift.

The above expression $\mathcal{J}$ represents the rate at which energy is emitted by the electron during the scattering process. It does not represent, however, the amount of energy scattered in a given direction because if a train of finite length is incident the length of the train reemitted in the direction $\theta$ is not necessarily the same. Since the number of waves in the incident and the scattered rays is the same, the ratio of the two lengths is the same as the ratio of the wave-lengths as measured in $K$ or the inverse ratio of the frequencies. Hence the amount of energy scattered is, using (4),

$$I_\theta = \frac{\nu_0}{\nu_{\theta c}}\mathcal{J}_\theta = \left(\frac{\nu_0}{\nu_\theta}\right)^2 \mathcal{J}_\theta . \tag{19}$$

We have, therefore, substituting (18),

$$I_\theta = I_{\theta c}\left[1+\frac{\nu_0}{N}(1-\cos\theta)\right]^{-4} . \tag{20}$$

Expression (20) represents the average rate at which energy is scattered by the electron if a number of trains of waves are incident, while (18) gives the rate of scattering during the incidence of one of the trains. Which of these expressions one must use depends upon the application. The important fact is that both $I_\theta$ and $\mathcal{J}_\theta$ are less than $I_{\theta c}[1+(\nu_0/N)(1-\cos\theta)]^{-3}$ and that the latter expression represents satisfactorily the experimental results. Expression (20) appears to be the correct one to use if the total amount of energy scattered is taken as the criterion. From that point of view very much more weight should be attached to the properties of the recoil electron than to those of the stationary one in calculations of intensity because the difference between $(1+y)^{-3}$ and $(1+y)^{-4}$ is smaller than between 1 and $(1+y)^{-3}$. Thus for small $y$

$$1-(1+y)^{-3} \cong 3y \text{ and } (1+y)^{-3}-(1+y)^{-4} \cong y$$

while for large $y$

$$1-(1+y)^{-3} \cong 1 \text{ and } (1+y)^{-3}-(1+y)^{-4} \cong 0$$

In justification of using expression (18), however, one may say that it deals with the stationary rate at which energy is scattered while a plane wave is continuously incident on the recoiling electron. Therefore, if the emitting atom and the recoiling electron are considered as a unit which is constantly radiating, expression (18) is the right one to use. It is a curious fact that the transition from (18) to the empirical formula

$$\bar{I}_\theta = I_{\theta c}\left[1+\frac{\nu_0}{N}(1-\cos\theta)\right]^{-3} \tag{21}$$

may be thought of as very analogous to the transition from $\nu_{\theta c}$ to $\nu_\theta$ represented by (4). We may say in fact that (21) is obtained from (18) by using the same intermediate value of the averaging parameter $x$ (see Eq. 7) as that which makes $\nu_{\theta c}$ go into $\nu_\theta$. In fact, this value is such that

$$\frac{1}{(1+a\nu_0 x)^2} = \frac{1}{1+a\nu_0 h}$$

and therefore

$$\frac{I_{\theta c}}{(1+a\nu_0 x)^6} = \frac{I_{\theta c}}{(1+a\nu_0 h)^3}$$

The meaning of this method of averaging is that the radiating ability of the atomic system is considered in such an intermediate stage that the classically computed frequency is the actual emitted frequency. It is of course known from the phenomena of dispersion that the reaction of an atomic system to a light wave depends on the properties of "virtual oscillators," and that the frequency of these oscillators is the quantum transition frequency. We attempt above to interpret this as meaning that approximately at least this is due to the action of an intermediate atomic system which is determined by that stage of the transition in which Bohr's corresponding frequency is equal to the emitted frequency.

It would be desirable, of course, to compare this method of averaging with others also in other cases. In view of the uncertainty of the atomic models, we cannot make a definite statement. However, it seems that the method used here is not worse than the customary ways of averaging and that it allows one to understand why in Lande's $g$-formula $j(j+1)$ takes place of $j^2$. A reason for this may be seen in the fact that different values of $j$ correspond to different $k$'s in the relativistic separation formula which is obeyed empirically with $j$ substituted for $k$. This means that the dependence of the energy level on $j$ is given by a term of the form $A/n^3j$ ($A$ a constant) and therefore the "corresponding" frequency is such that the value of $j$ which corresponds to it is given by

$$\frac{1}{c^2} = \frac{1}{j} - \frac{1}{j+1}$$

This means that $j_c^2 = j(j+1)$. Our assumption is here that the interaction between parts of an atom is not determined by the actual state of the atom but by a state intermediate between the state and a neighboring one, this intermediate state being chosen in such a way that the orbital frequency is equal to the frequency which would be emitted in the transition between the two states.

## 4. Comparison with Experiment

A comparison with experiment cannot be carried out satisfactorily at present on account of the scarcity of experimental material. The measurements on the Compton effect for x-rays are not covered by the above theory on account of the influence of atomic structure on the phenomenon of scattering. This question is treated in various papers by Jauncey.[3] We must rely, therefore, on the measurements of A. H. Compton[4] in the range of hard $\gamma$-rays. In these the wave-length of the scattered $\gamma$-rays has not been determined, and it is not known whether all of the scattered radiation was modified. Compton determined intensities of scattering in various directions relatively to scattering at 90°. He also found that $I_{90°}/I_0 = 0.037$. In the adjoining table the first column gives the angle of scattering, the second the average of Compton's experimental relative values, the third these values multiplied by 0.037, and the fourth values computed from our formula (21). The numbers in the fourth column are higher than those in the third. The fifth column gives ratios of numbers in the fourth to those in the third. The average of these is 1.3. It is striking that this is the ratio of 0.048 to 0.037. If, therefore, on account of a systematic error Compton's determination for 90° should be in error to that degree, formula (21) may be considered as representing experiments satisfactorily.

Table II

| $\theta$ | Average Relative Value | $0.037\chi$ Av. Rel. Value | Theoret. $I_{\theta c}(\frac{\nu_\theta^3}{\nu_0})$ | $\frac{\text{Theor.}}{\text{Exp.}}$ |
|---|---|---|---|---|
| 30° | 10.5 | 0.389 | 0.555 | 1.4 |
| 45 | 6.87 | 0.254 | 0.305 | 1.2 |
| 60 | 4.3 | 0.159 | 0.156 | 1.0 |
| 75 | 1.97 | 0.073 | 0.080 | 1.1 |
| 90 | 1.0 | 0.037 | 0.0476 | 1.3 |
| 120 | 0.7 | 0.0259 | 0.0288 | 1.1 |
| 135 | 0.45 | 0.0166 | 0.0267 | 1.6 |
| 150 | 0.47 | 0.0172 | 0.0260 | 1.5 |

[3] Jauncey, Phys. Rev. **25**, 314, 723 (1925); Phil. Mag. **49**, 427 (1925).
[4] Compton, Phil. Mag. **46**, 897 (1923).

It must be emphasized, however, that a definite conclusion may not be drawn at present because it is not known whether the unmodified radiation was entirely absent in Compton's experiments. Even a small amount of it would change the interpretation of the experiments because

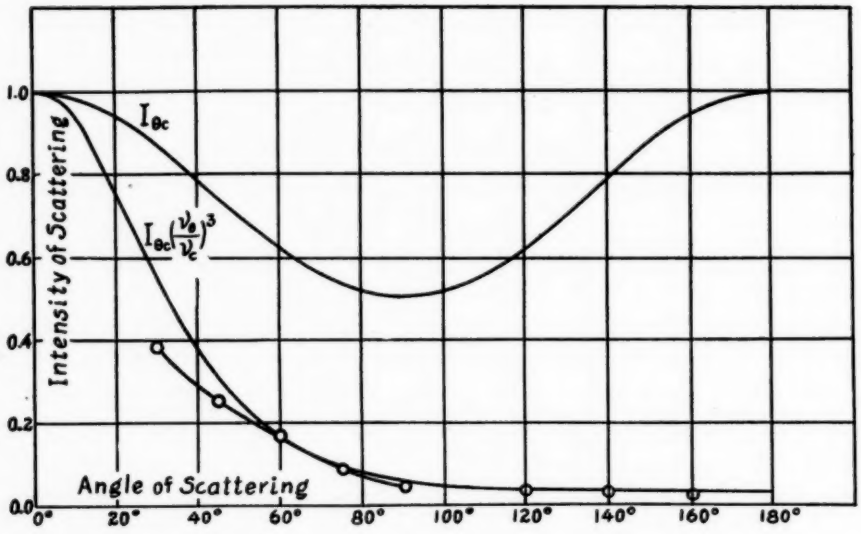


Fig. 3.

its dependence on the angle of scattering is very different from that given by (21). In Fig. 3 the J. J. Thomson formula is represented by $I_{\theta c}$, our (21) by $I_{\theta c}(\nu_\theta/\nu_0)^3$ and Compton's experimental values by the small circles.

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# AN EFFECT OF TEMPERATURE ON X-RAY ABSORPTION

By H. S. Read

Abstract

**The effect of temperature on the average atomic absorption coefficients for x-rays.**—The variation with temperature of the absorption by sheets of Al, Cu, Fe, Ni, Ag, and Pb of the total x-radiation from a tungsten x-ray tube operated at 50 kv has been studied. The measurements were made by balancing the ionization currents produced by two x-ray beams from the same tube, one of which passed through the absorber. After proper corrections are made for changes in the density of the absorber and of the air in the path of the beam, there remains a residual effect indicating a true variation of the atomic absorption coefficient with temperature. The results indicate that for all absorbers used the average atomic absorption coefficients for the total x-radiation from the tungsten target increased nearly linearly with the temperature by about 0.2 percent per 100°C up to temperatures near the melting point of the absorber. The work is being continued to determine the magnitude of the effect at different wave-lengths.

THE statement has been made,[1,2,3] and is generally accepted, that heating a crystal affects measurably its ability to reflect x-rays. Also it has been stated[4] that x-ray absorption is independent of all physical conditions, and specifically, independent of temperature. The critical analysis of x-ray phenomena show such close relations between transmission, reflection, scattering, and absorption that they appear to be consequences of one or two more fundamental phenomena. The object of the present work was to search for an effect of temperature on x-ray absorption by the metals, Al, Fe, Ni, Cu, Ag, and Pb.

A small temperature effect has been found and measured. Although this preliminary work does not give the accurate value of the small effect, it shows the existence of the temperature effect and the experiments are being continued to determine the value for the different wave-lengths.

## Method and Apparatus

A balance method of measurement was used. Two beams from one Coolidge tungsten x-ray tube (Fig. 1) were isolated by four pairs of lead slits and a Bumstead electroscope indicated their balance in two ionization chambers containing methyl bromide. An absorber in one beam was

[1] W. H. and W. L. Bragg, X-rays and Crystal Structure, 1st ed., pp. 190, 196.

[2] C. G. Darwin, Phil. Mag. **27**, 325 (1914).

[3] P. Debye, Ann. d. Physik **43**, 49 (1914); Vehr. d. D. Phys. Ges. **15**, 678, 738, 857 (1913).

[4] W. H. and W. L. Bragg, X-rays and Crystal Structure, 1st ed., p. 46.

heated by raising up an electric furnace with a narrow slot for the absorber holder. The furnace was 20 cm long, 8 cm inside diameter, and to prevent air currents was closed at the ends by Al sheet .001 cm thick. After raising the furnace the balance was found to be destroyed and the small amount that the adjustable slit in the other beam was changed in order to regain a balance, was recorded. All records and computations were made in terms of the adjustable slit width. Seven or nine trials, alternately hot and cold, were made and the difference between the averages computed. This difference was the uncorrected measure of the temperature effect.

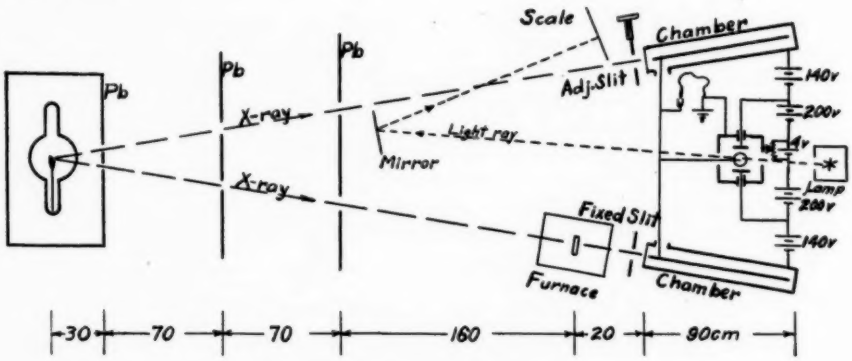


Fig. 1. Arrangement of apparatus.

Six 100 ampere-hour storage batteries were soldered together for a 12 volt source for the tube filament. Over one hour was allowed for the battery to approach a constant discharge current. After that time the tube electron current decreased very steadily at a rate of nearly 0.2 milliamperes per hour. A source of alternating current was obtained by running a compound wound rotary converter on a Terrill regulated d.c. generator. Voltage fluctuations larger than one percent would have been detected on the voltmeter.

The steadiness of operation is measured by the facts that the adjustable slit could be set to $1.0\times10^{-4}$ inch, and when set the electroscope deflection did not change over 1 mm in 10 seconds. However, some unknown cause gave variations as large as $30\times10^{-4}$ inch from day to day. The total absorption of the metals used was equivalent to 6000 to $9000\times10^{-4}$ inch and the temperature effect was from 0 to $200\times10^{-4}$ inch. The maximum slit opening was 1.2400 inch.

An improved Bumstead electroscope was used to indicate the balance between the ionization currents in the two chambers. The sensitivity used was about 150 mm per volt of leaf potential. The possible greater

sensitivity[5] would not have increased the accuracy proportionately because errors in single slit settings were smaller than other errors.

A correction for the smaller density of hot air in the x-ray path and for the Al furnace windows was made by measuring the effect of raising the furnace up around the empty holder while the absorber was across the x-ray beam but beyond the furnace. This was done for each absorber at each temperature used and the correction obtained was appropriate for the particular x-rays transmitted through the sample and for the particular furnace temperature. Since the order of position of successive, relatively thin absorbers does not affect much the total absorption, the different location of the absorber for the air correction observations caused no error.

Switching on and off the full load current in the furnace did not give any observable unbalance and consequently it was assumed that the fields of the furnace current did not cause any of the effect found.

The temperatures of the absorbers were measured in arbitrary units by a chromel-alumel thermocouple inside and near the wall of the furnace. Later this couple was calibrated in terms of a standardized Pt-Pt Rh couple, one junction of which was placed between two similar layers of the various absorbers.

## Effect of Heat Expansion

The measured change of x-rays transmitted was an absorption and a scattering effect. The latter is not easily computed and was not measured. However it probably was smaller than the absorption effect. For example Kaye[6] says that for the copper group the scattered is sometimes less than 1/200 of the total radiation and A. H. Compton[7] shows that scattering of other elements is of the same order of magnitude or less. And, since the absorber in the furnace covered the beam there and had about twice the area of the final slit the scattering into the beam partly compensated scattering out of the beam. No correction was made for scattering or reflection.

A correction was made for the thermal expansion of the absorber, an appreciable part of which expanded out of the path of x-rays. The beam was defined by a fixed slit, the projected area of which, at the absorber was an area $B$. Let $n$ represent the number of atoms per cc in the cool absorber, $x$ the cool absorber thickness, $a$ and $a'$ the average atomic ab-

[5] K. Cole has recently calibrated a Bumstead electroscope up to 37,000 mm per volt. J. Optical Soc. of America **10**, 99 (Jan. 1925).

[6] Kaye, X-rays, 4th ed., p. 114.

[7] A. H. Compton, National Research Council Bulletin **4**, Part 2, No. 20, p. 5 (1922).

sorption coefficients for the absorber cool and hot respectively, $I_0$ the incident intensity, and $I$ the transmitted intensity. For room temperature the transmitted energy is

$$W_1 = BI = BI_0 \exp\,[-anx] \tag{1}$$

and for the heated absorber is

$$W_2 = BI_0 \exp\left[-a'\frac{n}{(1+g)^3}(1+g)\right] = BI_0 \exp\left[\frac{-a'nx}{(1+g)^2}\right] \tag{2}$$

where $(1+g)$ is the usual thermal expansion coefficient for the absorber, values of which for the temperature ranges used were obtained from tables of Landolt and Bornstein, 5th edition, p. 1228. The percentage increase in transmitted energy due to heating is given by

$$K_c = \frac{W_2 - W_1}{W_1} = \exp\left[anx - \frac{a'nx}{(1+g)^2}\right] - 1\,. \tag{3}$$

Since the largest value of $2g$ used was .0388 and of $anx$ was 1.69 this expression reduces approximately to

$$K_c = (a-a')nx + 2ga'nx \tag{4}$$

TABLE I

*Computed results*

| | | | 20° | 190° | 320° | 475° | 630° | 880° | 1080° |
|---|---|---|---|---|---|---|---|---|---|
| Al | $anx$ | : | .66 | | | | | | |
| .102 cm | $K_m$ | : | .0003 | .0062 | .0035 | .0060 | .0085 | | |
| thick | $K_c$ | : | .0000 | .0054 | .0102 | .0195 | .0235 | | |
| | $K_m - K_c$ | : | 0003 | .0008 | −.0060 | −.0135 | −.0150 | | |
| Fe | $anx$ | : | .914 | | | | | | |
| .014 cm | $K_m$ | : | .0000 | .0025 | .0043 | .0050 | .0075 | .0134 | |
| | $K_c$ | : | .0000 | .0038 | .0073 | .0117 | .0164 | .0254 | |
| | $K_m - K_c$ | : | .0000 | −.0013 | −.0030 | −.0067 | −.0089 | −.0120 | |
| Ni | $anx$ | : | 1.14 | | | | | | |
| .008 cm | $K_m$ | : | .0014 | .0023 | .0046 | .0071 | .0106 | .0089 | .0190 |
| | $K_c$ | : | .0000 | .0053 | .0103 | .0150 | .0214 | .0319 | .0410 |
| | $K_m - K_c$ | : | .0014 | −.0030 | −.0057 | −.0079 | −.0108 | −.0408 | −.0220 |
| Cu | $anx$ | : | 1.14 | | | | | | |
| .009 cm | $K_m$ | : | −.0002 | .0024 | .0051 | .0078 | .0068 | .0023 | |
| | $K_c$ | : | .0000 | .0066 | .0119 | .0187 | .0258 | .0386 | |
| | $K_m - K_c$ | : | −.0002 | −.0065 | −.0068 | −.0109 | −.0190 | .0363 | |
| Ag | $anx$ | : | .95 | | | | | | |
| .005 cm | $K_m$ | : | .0002 | .0023 | .0054 | .0087 | .0117 | .0171 | |
| | $K_c$ | : | .0000 | .0063 | .0112 | .0176 | .0245 | .0368 | |
| | $K_m - K_c$ | : | .0002 | −.0040 | −.0058 | −.0089 | −.0128 | −.0197 | |
| Pb | $anx$ | : | 1.69 | | | | | | |
| .004 cm | $K_m$ | : | .0019 | .0115 | .0169 | | | | |
| | $K_c$ | : | .0000 | .0166 | .0301 | | | | |
| | $K_m - K_c$ | : | .0019 | −.0051 | −.0132 | | | | |

The atomic absorption coefficient is not generally assumed to change with any physical or chemical condition but the measurements reported here indicate that it is approximately proportional to the temperature. To show this first assume $a = a'$, giving $K_c = 2ganx$. Because $K_c$ is positive the transmission should be greater when the absorber is hot. The observed values, $K_m$, of the percentage increase in transmitted x-rays due to heating the absorber together with the values, $K_c$, calculated on the assumption that the change is due to the usual thermal expansion only, are given in Table 1 and are presented graphically in Fig. 2.

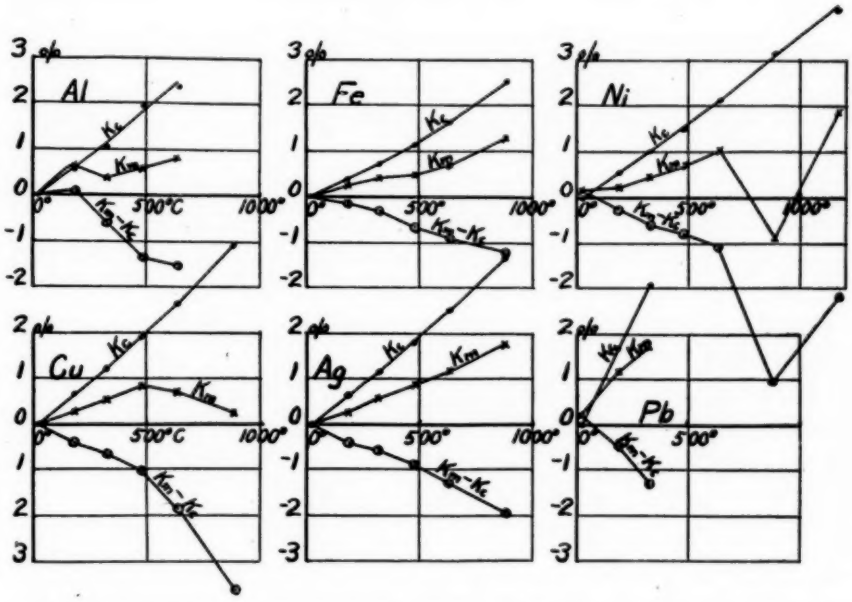


Fig. 2. Variation with temperature of the percent increase in transmission. $K_m$ is observed and $K_c$ calculated on the assumption that there is no variation of the atomic absorption coefficient with temperature.

To express the percentage decrease in transmission in terms of the percentage increase in absorption $(L_m - L_c)$ simply multiply by the ratio of transmitted to absorbed energy.

$$(L_m - L_c) = \frac{W_1}{W_0 - W_1}(K_m - K_c) \tag{5}$$

It will be noted that the measured change $K_m$ was smaller than the calculated change $K_c$. In fact $K_c$ was approximately twice $K_m$, except for a few temperatures. Therefore $a'$ is not equal to $a$. These results suggest that the average atomic absorption coefficient depends on the

temperature of the absorber in the following manner. Within the experimental error and for most of the temperatures and metals used

$$K_m = ganx \tag{6}$$

The true $K_e$ given by Eq. (4) should equal $K_m$

$$ganx = (a-a')nx + 2ga'nx \tag{7}$$

The largest value of $g$ was .0194 which may be neglected here when added to unity and Eq. (7) reduces approximately to

$$a' = (1+g)a\ . \tag{8}$$

This relation may be very important in the application of x-ray data to atomic structure studies. It should be confirmed by independent further measurements but unfortunately x-ray intensity measurements cannot, in the present state of the art, be measured much more accurately than was done for this work. The art of making accurate intensity measurements should be developed or new methods used for such work. Absorption coefficients are now known to three significant figures and as soon as they are measured to four figures a temperature correction will be necessary. It must be remembered that the $a$ used above is not the usual coefficient, for $a$ has widely different values for different wave-lengths. Only an effective value is used here. The work is being continued using narrow bands of wave-lengths and the smaller intensity of reflected x-rays makes it more difficult but not impossible to get sufficiently accurate results to show the temperature effect on absorption.

## Sample of Data and Computation

A complete set of observations and computations is given below:

Primary, 61.0 volts, 2.8 amps.; tube current, $3.1 \pm .1$ m.amps.; tube filament current, 3.75 amps.; electroscope sensitivity, 173 mm per volt; temperature in the furnace, 475°C.

Run 1. Air effect, x-rays filtered through the Ag strip.

| | | | | | Ave. | Diff. |
|---|---|---|---|---|---|---|
| Slit reading (cold): | .7812″ | .7806″ | .7800″ | .7797″ | .7804″ | |
| | | | | | | +.0072 |
| Slit reading (hot): | .7882 | .7875 | .7870 | | .7876 | |

Run 2. Ag absorber (0.5 min. allowed to heat or cool off)

| | | | | | | |
|---|---|---|---|---|---|---|
| Slit reading (cold): | .7811 | .7897 | .7798 | .7801 | .7801 | |
| | | | | | | .0032 |
| Slit reading (hot): | .7834 | .7832 | .7832 | | .7833 | |

The difference, $W_2 - W_1 = .0072'' - .0032'' = .0040''$ of the adjustable slit measures the effect of mass expanding out of the beam and any change

in absorption coefficient. $(W_2-W_1)/W_1$ is the measured percentage increase in transmission due to the heating.

No absorber in balanced a slit setting of .0405''. Complete absorption by a strip of Pb balanced 1.2400''. The $W_0$ incident on the absorber was $1.2400-.0405=1.1995$. The Ag transmitted

$$W_1=1.2400-.7804=.4596'' \tag{9}$$

Thus

$$K_m=\frac{W_2-W_1}{W_1}=\frac{.0040}{.4596}=.0087 \tag{10}$$

$$=0.87\% \text{ change .}$$

Since

$$\frac{I_0}{I}=\frac{1.2400-.0405}{1.2400-.7804}=2.61$$

$$anx=2.30\log\frac{I_0}{I}=0.95 \tag{11}$$

and, since for Ag $2g=16.8\times10^{-3}$,

$$K_c=2ganx=.0176 \tag{12}$$

$$K_m-K_c=-.0087+.0176=+.0089 \tag{13}$$

The Ag transmitted .89 percent less at 475°C than at 20°C or by Eq. (5) the Ag absorbed

$$\frac{.4596}{.7804-.0405}0.89\%=0.53\%$$

more at 475°C than at 20°C.

## Criticism

The increase in absorption due to heating was somewhat of a surprise and it seems appropriate to criticise the method and results as follows. The balance method is good for such work but the two x-ray beams were not exactly alike, and the absorber filtered one beam.

Total radiation used had a limiting wave-length of about 0.25A. The value of $anx$ used in computing $K_c$ is only an effective value and the approximation

$$\exp[2ganx]=1+2ganx \tag{14}$$

may not be close enough for all wave-lengths present. If the result, $K_m-K_c$, were small compared to $K_m$ more terms of the expansion should be used.

The value of $K_c$ depends on the expansion coefficients but it is improbable that there is the 50 percent error in the latter which would be needed to make the temperature effect zero.

The absorber and the absorber holder must have warped slightly on heating. To calculate the angle of warping which would just account for the effect simply remember that approximately $K_m = 2/3K_c$ and that $K_c$ is about 3 percent at 1000°. Thus an increase of 1 percent in the transmitted intensity when hot would make $K_m = K_c$. This requires 1.7 percent increase in $anx$ for Al, 0.7 percent for Pb, and intermediate values for the other metals. A warping of 8° would increase $x$ by 1 percent but that much of an angle seems improbable. No warping was observed.

No way was found to separate the effects of temperature on absorption, reflection and scattering; all are combined here.

The Braggs[8] show that the small amount of x-rays reflected from a crystal changes measurably with temperature of 370°. Collins[9] says that temperature has a marked effect on the scattered x-rays. It is suggested but not proved that the measured effect is larger than all the x-rays reflected and scattered and thus is larger than any probable change in reflection and scattering.

I wish to express my thanks to Professor F. K. Richtmyer for his pleasant interest and very generous aid. He suggested this study.

ROCKEFELLER HALL,
CORNELL UNIVERSITY,
February, 1924.

[8] W. H. and W. L. Bragg, X-rays and Crystal Structure, 1st ed., p. 196.
[9] E. H. Collins, Phys. Rev. **23**, 105 (1924).

# NOTE ON CRYSTALLINE AND AMORPHOUS STATES IN THE ALKALI METALS

By C. C. Bidwell

Abstract

X-ray diffraction patterns obtained on a wire of metallic lithium at 20°, 80°, 110°C show a decreasing diffracting power at the higher temperatures. This is consistent with the observations of Hull and McKeehan on sodium and potassium. The obliteration of the lattice structure at the higher temperatures is correlated with changes in the thermo-electric power and resistance lines previously found by the author for all the alkali metals.

IN a previous paper[1] on the resistance and thermo-electric power of the alkali metals a modification or structural change of some sort was shown to occur in the case of each of these metals, the transition being indicated by gradual changes in the resistance and thermo-electric lines. For lithium the change began to show at about $+50°$C, for sodium at about $-40°$C, for potassium about $-120°$C. The thermo-electric lines are straight to the transition temperature at which a gradual change in slope occurs. The resistance lines are of the form $R=R_0(1+\alpha t+\beta t^2)$ with different values for $\alpha$ and $\beta$ after the transition temperatures are passed.

A careful study of the crystal structure of lithium at various temperatures by x-ray diffraction methods indicates that the change is not a change from one crystalline pattern to another but a gradual disintegration of the lattice with rise of temperature. For this study a squirted wire of lithium was sealed into a thin walled glass tube, one-half of which was filled with sodium chloride and flour as a comparison control. The specimen was mounted vertically in the x-ray beam and continuously rotated about its axis of cylinder. Heating was effected by a stream of warm air passing through a paper cylinder which was slipped down over the specimen. By adjusting the velocity of the air stream and its temperature, any desired temperature could be maintained within five degrees.

At 20°C a 72 hour exposure gave 5 lithium lines and 13 strong lines for the NaCl control. The lithium lines were consistent with a body-centered cube as found by Hull.[2] At 83°C a 92 hour exposure gave three lines for lithium, and again the 13 strong lines for the NaCl control. At 110°C a 76 hour exposure still showed indications of the three lithium lines but

[1] Bidwell, Phys. Rev. **23**, 357 (1924).
[2] Hull, Phys. Rev. **10**, 661 (1917).

very much fainter than in the 83° exposure, while the NaCl control showed 14 lines indicating certainly no diminution in the intensity of the x-ray beam. New specimens were mounted up for each of these exposures. While the films indicate some persistence of the lattice at 110° they indicate certainly that the structure is much less definite at 110° than at 83° and less definite at 83° than at 20°.

Hull found difficulty in getting a sample of sodium which was not amorphous at room temperature but finally succeeded in getting a pattern of seven lines corresponding to a body-centered cube. He says "the tendency to form this regular arrangement is however very slight, corresponding to a small difference between the potential energies of the crystalline and amorphous states." McKeehan[8] was able to obtain a definite crystal pattern for potassium at −150°C and states that the observed crystalline structure does not persist when the temperature is allowed to rise again to 20°C.

It is the purpose of this note to suggest that the changes indicated by the resistance and thermo-electric lines for the alkali metals may be correlated with the gradual disintegration of the crystal lattice. The thermo-electric power is particularly affected, the slopes of these lines even changing in some cases from positive to negative. The fact that these curves are not exactly reproducible on successive runs may be explained as due to the persistence in varying degree of the crystalline forms at the higher temperatures. The change in the resistance lines is not so marked out nevertheless is definite and in entire agreement as to the temperature above which the metal tends to become amorphous.

The author wishes to express his gratitude to Dr. W. R. Whitney for offering the facilities of the Research Laboratory of the General Electric Company for this work and to Dr. W. P. Davey for the use of the special x-ray equipment.

CORNELL UNIVERSITY.
December 17, 1925.

[8] McKeehan, Proc. Nat. Acad. Sci. 8, 8 (1922).

# A NEW KIND OF TEST OF THE CORRESPONDENCE PRINCIPLE BASED ON THE PREDICTION OF THE ABSOLUTE INTENSITIES OF SPECTRAL LINES

By Richard C. Tolman and Richard M. Badger

Abstract

The *correspondence principle* postulates a relation between the average rate at which energy is actually being discontinuously emitted by quantum transitions and the continuous rate of emission which would be calculated from the classical theory. The principle has hitherto been used to predict the *relative intensities* of spectral lines of similar origin. Since, however, the classical theory permits a claculation of absolute rates of energy emission, it should be possible to calculate from the correspondence principle the *absolute intensities* of spectral lines. In the present article, the correspondence principle is subjected for the first time to this new kind of absolute test, using Czerny's data on the intensities of the far infra-red lines Nos. 8, 9, and 10 in the pure rotational spectrum of the dipole HCl. Tests of this kind have not previously been easy to make owing to lack of information as to the experimental values of absolute intensities or lack of knowledge of the the atomic models involved. The agreement between experimental and calculated intensities is close enough greatly to increase our confidence in the correspondence principle, justify us in subjecting it to further tests, and lead us to hope that an exact quantitative formulation of the principle may be found.

## I. Introduction

THE correspondence principle postulates a relation between the actual intensity of the light discontinuously emitted by the transitions of atoms from one quantum state to another, and the intensity of the light which would be continuously emitted on the basis of classical electromagnetics by the motion of the electric charges composing the atoms. The exact quantitative formulation of this relationship has not yet been discovered, although the problem has been specially considered by Kramers,[1] Hoyt,[2] Van Vleck,[3] Tolman,[4] and others. Indeed except for the case of large quantum numbers it is not even certain thay any exact relationship necessarily exists. Nevertheless, the principle has proved of extraordinary value in ruling out the occurrence of quantum transitions when on the basis of the classical theory there would be no tendency

[1] Kramers, Kgl. Danske. Vidensk. Selsk. Skrifter **8, III,** 287 (1919).
[2] Hoyt, Phil. Mag. **46,** 135 (1923); *Ibid.* **47,** 826 (1924).
[3] Van Vleck, Phys. Rev. **24,** 334 (1924).
[4] Tolman, Phil. Mag. **49,** 130 (1925).

for the emission of radiation of the kind in question, and furthermore especially in the hands of Kramers (loc. cit.[1]) has permitted at least the approximate prediction of the relative intensities of a very considerable number of spectral lines. These successes of the correspondence principle make us hope that its range of application can be extended.

In all of the tests of atomic or molecular behavior to which the correspondence principle has hitherto been subjected it has only been feasible to compare the experimental facts either with the prediction that a given spectral line will not occur at all or with a prediction as to the *relative* intensities of related spectral lines. Nevertheless, since the classical theory provides us with a method of estimating the absolute rates with which energy is absorbed or emitted by moving charges, it should be possible to predict with the help of the correspondence principle the *absolute* intensity of absorption and emission lines.

It is the purpose of the present article to subject the correspondence principle to this new kind of more exacting test by comparing experimental and theoretical values for the strength of absorption lines of hydrogen chloride in the far infra-red. Such a test of predictions as to absolute intensities has not previously been possible, either because of the very limited number of spectral lines for which we know the absolute intensity, or because we did not have sufficient knowledge as to the atomic models involved. The recent beautiful work of Czerny,[5] however, on hydrogen chloride provides us with information as to the absorption of radiation accompanying changes in the velocity of rotation of a simple electric dipole, while the measurements of Zahn[6] coupled with the calculations of Pauling[7] give us a value for the electric moment of the dipole in question. As the result of the computations thus made possible, we find an agreement between experiment and theory, which is probably as close as could be expected with our present development of experimental technique and theoretical insight. This verification of a prediction as to absolute intensities cannot fail to increase our confidence in the correspondence principle, justify us in subjecting it to further tests and applications, and lead us to hope that the principle may sometime receive a correct and precise quantitative formulation.

## PART I. CALCULATION OF ABSORPTION COEFFICIENT

2. *Refinements in method of calculation.* Our first task will be to calculate from Czerny's absorption data the value of Einstein's coefficient

[5] Czerny, Zeits. f. Physik **34**, 227 (1925).
[6] Zahn, Phys. Rev. **24**, 400 (1924).
[7] Pauling, Proc. Nat. Acad. **12**, 32 (1926).

$B_{ij}$ which gives the probability of transition from a lower quantum state $S_i$ to a higher state $S_j$ by the absorption of radiation. We shall then compare the experimental value of $B_{ij}$ with that predicted by the correspondence principle.

The method of calculating values of $B_{ij}$ and the results obtained in a wide variety of cases are illustrated by an earlier paper of Tolman.[8] In these earlier calculations, however, the difference in the energy levels corresponding to states $S_i$ and $S_j$ was so large that the number of molecules in the upper state $S_j$ was relatively very small and the reinforcement of the primary beam by induced transitions from the upper to the lower state could be neglected. Our present treatment will be refined to cover this point, since the number of molecules in the upper quantum states under consideration is large enough so that the induced emission is in the neighborhood of fifty percent of the primary absorption.

The treatment will also be modified in the direction of greater precision by not introducing into the theoretical development the assumption that the effective width of the line is small as has hitherto been customary. In the numerical application of the theory to the data in hand, however, we shall find it possible to introduce this simplification.

3. *Einstein's treatment of absorption and emission.* Calculations of the kind to be made must be based on Einstein's 1917 formulation[9] of the nature of the absorptive and emissive processes. In accordance with this formulation, if we consider a system containing $N_i$ molecules in the lower quantum state $S_i$, and $N_j$ molecules in the upper state $S_j$, surrounded by radiation of the frequency $\nu$ involved in the quantum transitions, we may write the following expressions governing the rates of passage of molecules between the states by interaction with radiation. For the passage of molecules from state $S_i$ to $S_j$ by absorption we have

$$-dN_i/dt = N_i \rho_\nu B_{ij} \tag{1}$$

where $\rho_\nu$ is the density of radiation of the frequency $\nu$ which can be absorbed, and the constant $B_{ij}$ is Einstein's coefficient of absorption. And for the passage of molecules in the reverse direction, we may write

$$-dN_j/dt = N_j A_{ji} + N_j \rho_\nu B_{ji} \tag{2}$$

where the constant $A_{ji}$ is Einstein's coefficient of spontaneous emission and the constant $B_{ji}$ the coefficient of induced emission. Furthermore, as a result of Einstein's considerations we have the following relations connecting the coefficients

[8] Tolman, Phys. Rev. **23**, 693 (1924).

[9] Einstein, Phys. Zeits. **18**, 121 (1917).

$$A_{ji}=\frac{8\pi h\nu^3}{c^3}\frac{p_i}{p_j}B_{ij}\ ; \qquad B_{ji}=\frac{p_i}{p_j}B_{ij} \tag{3}$$

where $p_i$ and $p_j$ are the a priori probabilities of the two states. In applying the above equations to an absorption experiment to calculate the effect of the molecules of a medium on the transmission of a beam of light, it should be noted that each molecule which changes from the lower state $S_i$ to the upper state $S_j$ under the action of the light in accordance with Eq. (1) is to be regarded as abstracting a quantum of light $h\nu$ from the oncoming beam. On the other hand for the molecules which pass in the opposite direction in accordance with Eq. (2), a careful distinction must be drawn, since those dropping by spontaneous emission are to be regarded as giving off quanta $h\nu$ of random direction and phase, while those which go down by induced emission are to be thought of as giving quanta coherent with the stimulating radiation and hence reinforcing the primary beam.

4. *Modified formulation of absorption and emission.* The original formulation given above applies strictly only to a system in which the radiation that can be absorbed or emitted may be treated as having a single definite frequency $\nu$, a condition of affairs which corresponds on the experimental side to a spectral line having no appreciable width and on the theoretical side to a system in which the molecules in a given quantum state all have exactly the same energy content. In actual systems, however, molecules in a given quantum state do not all have the same energy content, but owing to the effect of fields from neighboring molecules, to the velocity of thermal motion, and perhaps in a small degree to a fundamental lack of sharpness, the molecules in a given quantum state have a range of energies distributed around a most probable value. And hence in accordance with the fundamental equation $E=h\nu$ we obtain a range of energy changes and corresponding range in frequencies for a given quantum transition. Hence since the spectral lines in the case which we are to treat are rather broad compared with their absolute position in the frequency scale, it will be desirable to modify the original Einstein formulation.

The possibility and nature of a new formulation which will allow for the absorption or emission of a continuous range of frequencies has already been indicated by Pauli[10] and by Einstein and Ehrenfest,[11] and it is believed that the following set of relations is in agreement with their point of view. In these new relations we shall use small letters $b_{ij}$, $a_{ji}$

[10] Pauli, Zeits. f. Physik **18**, 272 (1923).

[11] Einstein and Ehrenfest, Zeits. f. Physik **19**, 301 (1923).

and $b_{ji}$ for the probabilities of absorption and emission, and shall regard these quantities as continuous variables which may change as we pass across a spectral line. For any given frequency $\nu$ we shall then write for the rate of passage of molecules from the lower to the higher state

$$-\frac{d(\delta N_i)}{dt}=(\delta N_i)\rho_\nu b_{ij} \tag{4}$$

and for the passage in the opposite direction

$$-\frac{d(\delta N_j)}{dt}=(\delta N_j)a_{ji}+(\delta N_j)\rho_\nu b_{ji} \tag{5}$$

where

$$\delta N_i=\frac{\partial N_i}{\partial \nu}\delta\nu \text{ and } \delta N_j=\frac{\partial N_j}{\partial \nu}\delta\nu \tag{6}$$

are the numbers of molecules in the two quantum states which have exactly the right configuration to interact with radiation of frequency in the range $\nu$ to $\nu+\delta\nu$. Furthermore we shall also write the relations

$$a_{ji}=\frac{8\pi h\nu^3}{c^3}\frac{p_i}{p_j}b_{ij} \text{ and } b_{ji}=\frac{p_i}{p_j}b_{ij} \tag{7}$$

connecting the coefficients,—relations which are sufficient to guarantee agreement with the Planck radiation law.

We shall further find it convenient to retain the original Einstein coefficients $B_{ij}$, $A_{ji}$ and $B_{ji}$ by regarding them as average values for the spectral line under consideration in accordance with the equations of definition

$$B_{ij}=\frac{1}{N_i}\int b_{ij}\delta N_i \; ; \qquad A_{ji}=\frac{1}{N_j}\int a_{ji}\delta N_j \; ; \qquad B_{ji}=\frac{1}{N_j}\int b_{ji}\delta N_j \tag{8}$$

5. *Relation between the coefficient $B_{ij}$ and the absorption coefficient $a_\nu$.* We may now apply the foregoing considerations to the interpretation of an absorption experiment. Consider a beam of light of total intensity $I$ in ergs $cm^{-2}$ $sec^{-1}$, and let us define the intensity for a given frequency by the equation

$$I_\nu=dI/d\nu \tag{9}$$

Let the beam of light fall on a layer of absorbing medium having a cross section of one square centimeter and a thickness $dx$, and containing per cubic centimeter $\delta N_i$ molecules in a condition to absorb radiation in the frequency range $\nu$ to $\nu+\delta\nu$ and $\delta N_j$ in a condition to emit this fre-

quency. Since the density of radiation corresponding to the intensity $I_\nu$ is obviously given by the equation

$$\rho_\nu = I_\nu/c \tag{10}$$

where $c$ is the velocity of light, we may evidently write, in accordance with Eqs. (4) and (5), for the rates at which molecules in the layer of unit cross section and thickness $dx$ are passing from one condition to the other by absorption and induced emission the expressions

$$-\frac{d}{dt}(\delta N_i dx) = (\delta N_i dx)\frac{I_\nu}{c} b_{ij} \text{ and } -\frac{d}{dt}(\delta N_j dx) = (\delta N_j dx)\frac{I_\nu}{c} b_{ji} \tag{11}$$

For each molecular transition, however, one quantum of energy $h\nu$ is either absorbed or returned to the beam, so that we easily obtain from Eqs. (11) an expression for the net rate at which energy in the frequency range $\nu$ to $\nu+\delta\nu$ is absorbed from the beam because of its passage through the layer, and can write

$$-d(I_\nu \delta\nu) = (\delta N_i dx)\frac{I_\nu}{c} b_{ij} h\nu - (\delta N_j dx)\frac{I_\nu}{c} b_{ji} h\nu \tag{12}$$

By the rearrangement of (12) and introduction of the expression for $b_{ji}$ given by (7) we obtain

$$-\frac{1}{I_\nu}\frac{dI_\nu}{dx} = \frac{h\nu b_{ij}}{c}\left(\frac{\delta N_i}{\delta\nu} - \frac{p_i}{p_j}\frac{\delta N_j}{\delta\nu}\right) \tag{13}$$

and this is evidently the ordinary absorption coefficient for light of frequency $\nu$ which may be appropriately designated by the symbol $a_\nu$ giving us

$$\frac{h\nu b_{ij}}{c}\left(\frac{\delta N_i}{\delta\nu} - \frac{p_i}{p_j}\frac{\delta N_j}{\delta\nu}\right) = a_\nu \tag{14}$$

If now we assume a condition of thermal equilibrium we may write in accordance with the Maxwell-Boltzmann distribution law

$$\delta N_j = \delta N_i \frac{p_j}{p_i} e^{-h\nu/kT} \tag{15}$$

and by substitution and rearrangement obtain in accordance with the definition of $B_{ij}$ given by (8)

$$B_{ij} = \frac{1}{N_i}\int b_{ij}\delta N_i = \frac{c}{hN_i}\int \frac{a_\nu d\nu}{\nu(1-e^{-h\nu/kT})} \tag{16}$$

where the integration is to be taken across the effective width of the absorption line. This would seem to be an exact expression for $B_{ij}$ which is the average value of the coefficient $b_{ij}$, and is the quantity to be compared with that obtained from the correspondence principle.

6. *Calculation of $B_{ij}$ from Czerny's results.* In the far infra-red Czerny (loc. cit.[5]) has measured the absorption of hydrogen chloride gas corresponding to simple changes in the state of rotation of the electric dipole $H^+Cl^-$. In particular he gives a plot of percentage transmission as a function of wave-length in the region of lines 8, 9, and 10 which correspond to changes in the rotational quantum number $m$ of ($7\frac{1}{2}$ to $8\frac{1}{2}$), ($8\frac{1}{2}$ to $9\frac{1}{2}$) and ($9\frac{1}{2}$ to $10\frac{1}{2}$), and although he states that exact information as to the intensities of absorption and width of line cannot be taken from the plot owing to the relatively great width of his spectrometer slit, nevertheless his results would seem to be approximately reliable. Furthermore, he states that the length of his absorption tube was 12 cm, while for the three lines in question he appears to have used hydrogen chloride gas at one atmosphere pressure and room temperature. We thus have for the three lines mentioned all the data necessary for substitution in Eq. (16).

Since, even for these lines in the far infra-red, the effective width of the lines is not great compared with their absolute position in the frequency scale, we can rewrite Eq. (16) for our purposes in the approximate form

$$B_{ij}=\frac{c}{hN_i\nu_0(1-e^{-h\nu_0/kT})}\int a_\nu d\nu \tag{17}$$

where $\nu_0$ is the frequency of the center of the line.

The number of molecules $N_i$ in different states of rotation corresponding to different rotational quantum numbers

$$m=1/2, 3/2, 5/2, \ldots (i-1/2) \tag{18}$$

can be calculated in accordance with the Maxwell-Boltzmann distribution law from the equation[12]

$$N_i=\frac{Np_ie^{-\epsilon_i/kT}}{\sum_i p_ie^{-\epsilon_i/kT}}=N\frac{2ie^{-(i-\frac{1}{2})^2h^2/8\pi^2JkT}}{\sum_i 2ie^{-(i-\frac{1}{2})^2h^2/8\pi^2JkT}} \tag{19}$$

and for high temperatures may introduce the approximation

$$\sum_i 2ie^{-(i-\frac{1}{2})^2h^2/8\pi^2JkT}=\sum_i 2ie^{-(i-\frac{1}{2})^2\sigma}\approx(1/\sigma)[1+\tfrac{1}{2}\sqrt{\pi\sigma}] \tag{20}$$

[12] See Tolman, Phys. Rev. **22**, 470 (1923).

where $N$ the total number of molecules per cc at one atmosphere and 20°C, can be taken as $2.52\times10^{19}$ and the term containing $J$, the moment of inertia of the molecule can be taken from Czerny's own work which gives $h/8\pi^2 J = 10.397\ c$. The results for the three lines nos. 8, 9, and 10 are given in column two of Table I.

Values for the wave numbers $\nu_i/c$ corresponding to the centers of the lines were obtained by Czerny as given in column three, and values of the factor $(1-e^{-h\nu_0/kT})$ have been calculated and are given in column four of Table I.

Finally the values of the integral $\int a_\nu d\nu$ have been obtained from Czerny's plot of percentage transmission $I/I_o$ as function of wave-length $\lambda$ by replotting $\log I/I_o$ against $1/\lambda$ and with the help of a graphical integration substituting into the equation

$$\int a_\nu d\nu = -\frac{2.30c}{x}\int \log\frac{I}{I_0}\,d\left(\frac{1}{\lambda}\right) \tag{21}$$

where for $x$ we use the length of the absorption tube 12 cm. The values obtained are given in column five of Table I.

We now have all the quantities necessary for substitution into Eq. (17) and obtain for $B_{ij}$ the results given in the last column of Table I.

TABLE I

(c. g. s. units)

| No. of line | $N_i$ | $\nu_0/c$ | $(1-e^{-h\nu_0/kT})$ | $\int a_\nu d\nu$ | $B_{ij}$ |
|---|---|---|---|---|---|
| 8 | $0.9806\times10^{18}$ | 165.63 | 0.5553 | $3.318\times10^{10}$ | $5.61\times10^{16}$ |
| 9 | $4.897\times10^{17}$ | 185.86 | 0.5970 | $0.425\times10^{10}$ | $6.81\times10^{16}$ |
| 10 | $2.180\times10^{17}$ | 206.38 | 0.6356 | $1.987\times10^{10}$ | $10.59\times10^{16}$ |

PART II. THEORETICAL CALCULATIONS OF THE ABSORPTION COEFFICIENTS

7. *Application of the correspondence principle to a dipole rotator.* We must now turn our attention to the theoretical estimation of the above absorption coefficients with the help of the correspondence principle, which postulates a relation between the rates at which energy is actually emitted by molecules in accordance with the quantum theory and the rates which would be calculated on the basis of the classical theory.

Consider a system containing a number of dipole rotators all in the same quantized state of rotation $S_j$ corresponding to the rotational quantum number $m_j$. It we treat the system as degenerate, i.e. not spatially quantized, the axes of rotation of these dipoles will be oriented in random directions, and the emission of radiation will result from a jump to a lower state of rotation $S_i$ corresponding to the rotational quantum number $m_i = m_j - 1$, the axis of rotation for any particular dipole retaining its direction in space.

On the basis of the *quantum theory* the average rate of emission of energy per rotator will evidently be

$$\left(\frac{dR}{dt}\right)_Q = A_{ji}h\nu \tag{22}$$

where $A_{ji}$ is Einstein's coefficient of spontaneous emission.

On the other hand, on the basis of the *classical theory* the rate of spontaneous emission of energy by a dipole rotator would be

$$\left(\frac{dR}{dt}\right)_{Cl} = 2\times\frac{16\pi^4}{3c^3}C^2\omega^4 \tag{23}$$

where $C$ is the electric moment of the dipole and $\omega$ the frequency of revolution.

The *correspondence principle* postulates now the existence of a relation between such pairs of expressions for the rates of energy emission, as given by the two theories. As already stated the exact formulation of this relation if it exists has not yet been discovered. It seems reasonable, however, to presume that the correct quantum theory rate of emission as given by Eq. (22) might be determined by some method of averaging the classical rates which in accordance with Eq. (23) would be calculated for the initial and final states of the rotator or perhaps for the mechanically possible intermediate states. This presumption can be symbolically expressed by the equation

$$\frac{dR}{dt} = A_{ji}h\nu = 2\times\frac{16\pi^4}{3c^3}[C^2\omega^4]_{av} = 2\times\frac{16\pi^4}{3c^3}Q^2\nu^4 \tag{24}$$

where we have introduced for convenience a new quantity $Q$ which may be regarded as the electric moment of an idealized classical rotator having the frequency $\nu$ and desired rate of energy emission $A_{ji}h\nu$.

8. *Methods of averaging.* The method of averaging, which has received the most study and in the hands of Kramers (loc. cit.[1]) has led to excellent estimates of the relative intensities of lines, would either make the hypothetical electric moment $Q$ merely the mean of the actual electric moments in the initial and final quantum states in accordance with the equation

$$Q = \frac{C_i + C_j}{2} \tag{K}$$

or would make the square of $Q$ the mean of the squares of $C_i$ and $C_j$ in accordance with the equation

$$Q^2 = \frac{C_i^2 + C_j^2}{2} \tag{K}$$

Six methods of averaging over the mechanically possible intermediate states, one of which was proposed by Kramers, have also been studied by Hoyt (loc. cit.[2]) and by Tolman (loc. cit.[4]) and are represented by the following equations

$$Q^2 = \int_0^1 C^2 d\lambda \tag{A}$$

$$Q = \int_0^1 C\, d\lambda \tag{B}$$

$$\log Q = \int_0^1 \log C\, d\lambda \tag{C}$$

$$Q^2\nu^4 = \int_0^1 C^2\omega^4\, d\lambda \tag{D}$$

$$Q\nu^2 = \int_0^1 C\omega^2\, d\lambda \tag{E}$$

$$\log (Q\nu^2) = \int_0^1 \log (C\omega^2) d\lambda \tag{F}$$

where $\lambda$ is an auxiliary variable which is related to the phase integral ($I = \int pdq$) taken for intermediate states by the equation

$$I = I_i + \lambda(I_j - I_i) \tag{25}$$

Since for a *rigid* dipole the electric moment is a constant, four of the methods of averaging evidently lead to a simple equality of the hypothetical moment $Q$ and the actual moment $C$, as shown in Table II.

TABLE II

| Method of Average $K, A, B, C$ | Value of $Q^2$ for transition $(m+1)$ to $m$ $Q^2 = C^2$ |
|---|---|
| $D$ | $\frac{16}{5}\frac{(m+1)^5 - m^5}{(2m+1)^4} C^2$ |
| $E$ | $\frac{16}{9}\frac{\{(m+1)^3 - m^3\}^2}{(2m+1)^4} C^2$ |
| $F$ | $\frac{16}{e^4}\frac{(m+1)^{4(m+1)}}{m^{4m}(2m+1)^4}$ |

[13] See Tolman, *loc. cit.*, footnote 4. The treatment given in this article was worded so as to apply to a rigid dipole rotator with whole quantum numbers. The results quoted, however, are the same for the case of whole and half quantum numbers.

The results for the other three methods of averaging in terms of the rotational quantum numbers $m$ and $m+1$ of the two states have previously been published[13] and are also given in Table II.

Substituting the values for $m$ (7.5, 8.5 and 9.5), corresponding to the three lines under investigation, into the expression given by methods $D$, $E$, and $F$ of averaging, we find that these methods also give within one percent the same value

$$Q^2 = C^2 \tag{26}$$

and hence we shall use this in our following calculations. It is indeed fortunate that we have data to test so simple a case as that of a rigid dipole rotator, since it seems probable that for any reasonable method of averaging, expression (26) would give at least the right order of magnitude for $Q^2$.

9. *Relation between $B_{ij}$ and $Q^2$.* To calculate the theoretical value of Einstein's coefficient of absorption $B_{ij}$ from the value of $Q^2$ we have merely to combine Eqs. (3) and (24) in the text and obtain

$$B_{ij} = 2 \times \frac{2\pi^3}{3h^2} \frac{p_j}{p_i} Q^2 \tag{27}$$

10. *Numerical value of $Q$ and theoretical values of $B_{ij}$.* The measurements by Zahn[6] of the dielectric constant of hydrogen chloride gas make it possible to calculate the electric moment of the dipole and this has been done by Pauling[7] in accordance with the theory of half quantum numbers. He obtains the value $0.3316 \times 10^{-18}$ in C.G.S. electrostatic units.

Taking this as the value of $Q$ and substituting in Eq. (27) we obtain for $B_{ij}$ the values 10.9, 10.8, and $10.7 \times 10^{16}$ for the successive lines Nos. 8, 9, and 10.

11. *Application of correspondence principle treating the system as non-degenerate.* In the foregoing calculations, as noted above, we have treated the dipole rotators as degenerate systems, with their axes of rotation distributed in random directions. It has been suggested by Kemble,[14] however, in considering rotation-oscillation spectra, that the correspondence principle ought to be applied to such systems by treating them as though they had been made non-degenerate by spatial quantization in a weak external (magnetic) field.

Under these circumstances, the axes of rotation can only assume certain spatially quantized directions and we must use two quantum numbers to describe the state of the rotator, a rotational quantum number $m$

[14] Kemble, Phys. Rev. **25**, 1 (1925).

which is proportional to the total angular momentum of the rotator, and an equatorial quantum number $r$ which is proportional to the component of the angular momentum parallel to the applied field, the angle between the axis of rotation and the external field being given by the equation $\cos\theta = r/m$. The possible absorption processes may then be correlated with a change in the rotational quantum number from $m$ to $m+1$ and changes in the equatorial quantum number from $r$ to $r \pm 1, 0$. Furthermore each of these processes may be correlated with terms in the Fourier analysis for the motion of the rotator, and the frequency of occurrence of the different transitions estimated from the magnitude of the coefficients of the corresponding terms in the Fourier analysis.

We have also made an estimation of the intensities of the three lines in question on this basis. The calculation is complicated by the fact that the Fourier coefficients corresponding to a given quantum transition now have in general different magnitudes in the initial and final state of the molecule, since they depend on the angle between the external field and the axis of rotation and in general this changes when the transition occurs. In order to overcome this difficulty, following the procedure of Kemble, we have determined the Fourier analysis for an intermediate motion, in which the angle between the external field and axis of rotation is given by the equation $\cos\theta = \bar{r}/\bar{m}$, where $\bar{r}$ and $\bar{m}$ are the respective means of the values of the quantum numbers $r$ and $m$ in the initial and final state. We have then used the Fourier coefficients of the proper terms in the analysis of this intermediate motion as the amplitudes of the idealized classical motions of frequency $\nu$ which on the basis of the correspondence principle may be assumed to give the same rate of energy emission as the average rate given by the quantum transition in question.

We do not reproduce the calculations here, since they are somewhat lengthy, and for the particular absorption lines under consideration lead nearly to the same results as our previous simpler calculations which treated the rotators as degenerate systems. The approximate agreement of these new results obtained by treating the rotators as non-degenerate is due to the fact that the theory of spatial quantization prescribes no less than sixteen values of the angle between field direction and precessing axis even for the molecules of lowest energy content involved in the production of lines Nos. 8, 9, and 10, and the changes in direction of the axis accompanying the permitted transitions are small. Hence the spatially quantized system of rotators behaves nearly the same as a degenerate system in which the axes of rotation are uniformly distributed in direction and there is no change in direction when a quantum transition takes place. For the first lines in the rotational spectrum the two methods of

treatment lead to quite different results. For this reason an experimental determination of the intensities of these lower lines would be interesting.

12. *Newer methods of treatment.* In addition to the methods of applying the correspondence principle discussed above, there has recently been considerable investigation of the calculation of intensities by methods specially adapted to agree with the Burger Dorgelo "sum rule" for the intensities of the components of a multiplet line. These methods may be called with Fowler "refined applications" of the correspondence principle. In particular Professor J. H. Van Vleck has been good enough to call our attention by letter to an article by Reiche and Thomas[15] in which an expression has been obtained for the absolute intensity of the pure rotational lines for a dipole rotator. It is interesting to note that their expression (see page 521 l.c.) becomes identical with our equation (27) obtained from methods of averaging K, A, B, C, provided we modify their equation in such a way as to allow for the fact that the electric separation in the hydrogen chloride molecule is different from the internuclear distance, and further substitute the same a-priori weights which we have employed. Hence we may regard our calculations as also agreeing with the work of Reiche and Thomas.

It should also be noted that the new quantum mechanics of Born and Heisenberg is specially adapted for the calculation of intensities of lines, but we have not yet carried through a calculation along these lines. To do this, it would first be necessary to consider whether, by going over to the new basis, modifications might not have to be introduced into Pauling's calculation of electric moment.

## PART III. COMPARISON OF EXPERIMENTAL AND THEORETICAL VALUES OF $B_{ij}$

13. *Conclusion.* We may now compare the experimental values of $B_{ij}$ calculated from Czerny's absorption measurements with the theoretical values calculated from the correspondence principle, as given below in the last two columns of Table III.

TABLE III

| No. of line | Transition | $B_{ij}$(exp.) | $B_{ij}$(theor.) |
|---|---|---|---|
| 8 | $m = 7.5$ to $m = 8.5$ | $5.6\times10^{16}$ | $11.9\times10^{16}$ |
| 9 | $m = 8.5$ to $m = 9.5$ | $6.8\times10^{16}$ | $11.8\times10^{16}$ |
| 10 | $m = 9.5$ to $m = 10.5$ | $10.6\times10^{16}$ | $11.6\times10^{16}$ |

The agreement is not exact, and perhaps outside the experimental error of the two sets of measurements of Czerny and Zahn which were used in the calculations. Nevertheless when we consider the difficulties

[15] Reiche and Thomas, Zeits. f. Physik, **34**, 510 (1925).

of both kinds of experimental work and the complexity of the three calculations by which the electric moment was obtained from the measurement of dielectric constant, and Einstein's coefficient was calculated first from the absorption measurements and then from the correspondence principle, the results are certainly of a nature to increase our respect for the correspondence principle, which has now been subjected for the first time to this kind of *absolute* test, involving the absolute rather than the relative intensities of spectral lines.

The most striking characteristic of the lack of agreement is the gradual rise in the experimental values of $B_{ij}$ as we go to higher quantum numbers approaching perhaps the nearly constant value predicted from the correspondence principle. This may indicate the necessity of a progressive modification of the correspondence principle as we go to lower quantum numbers, a possibility which finds some support from other directions. Speculations, however, as to the precisely correct manner of applying the correspondence principle can best be postponed until more certain experimental data are at hand. An attempt to obtain further data for this purpose is already under way in this laboratory.

NORMAN BRIDGE LABORATORY OF PHYSICS,
PASADENA, CALIFORNIA,
January 19, 1926.

# THE ARC SPECTRA OF IODINE, BROMINE, AND CHLORINE IN THE SCHUMANN REGION

By Louis A. Turner

Abstract

The arc spectra of Cl, Br, and I have been investigated in the region between 2050 and 1230 A. U. Wave-length tables are given. 1396.5 and 1633.6 A. U. are the wave-lengths of the lines of longest wave-length of Cl and Br, respectively, and correspond to the previously known 2062.1 line of I. Doublet differences of 881, 3685, 7600 wave number units are found between lines of Cl, Br, and I spectra respectively and are thought to be the separations of the two levels of normal doublet $p$ states of the atoms. The probable resonance lines are of wave-lengths 1379.6, 1540.8 and 1782.9 A. U. corresponding to radiating potentials of 8.95, 8.01, and 6.92 volts. No band spectrum of HCl appeared in the region investigated although the excitation was probably favorable for its production.

SEVERAL investigators[1,2,3] have observed four very intense spectrum lines, emitted by mildly excited iodine vapor, in the region of the shortest wave-lengths which can be photographed with a quartz spectrograph. Their great intensity and the conditions for their excitation[3] suggested that these lines were probably arc lines arising from transitions between low energy levels of the atom, and that more strong lines which might show simple spectrum relationships would be found in the Schumann region. The study of this region of the spectrum was made possible by Professor T. Lyman who very kindly permitted me to use his vacuum spectrograph.

## Apparatus and Methods

The vacuum spectrograph has been fully described elsewhere.[4] The grating was one of 1 meter focus with 15,000 lines to the inch, ruled by Professor R. W. Wood at Johns Hopkins University. Experience has shown it to give spectra nearly free from ghosts of the Lyman type. A fluorite plate was placed over the slit to prevent the halogen vapors from entering the body of the spectrograph, consequently the shortest wave-length which could be measured was about 1230 A.U. The grating was so set that the longest wave-length observed was 2050 A. U. The

[1] E. B. Ludlam and W. West, Proc. Roy. Soc. Edinburgh **44 II,** 185 (1923–24).
[2] C. Füchtbauer and E. Holm, Phys. Zeits. **26,** 345 (1925).
[3] R. S. Mulliken and L. A. Turner, Phys. Rev. **25,** 886 (1925).
[4] T. Lyman, Astrophys. J. **50,** 1 (1924).

spectra were photographed in the first order, the dispersion being 16.84 A. U. per millimeter.

The discharge tube had tungsten wire electrodes and an internal capillary tube placed close to the slit (Fig. 1). For nearly all of the exposures a continuous current of 10–15 milliamperes supplied by an arrangement of a transformer, kenetron rectifiers, and a large condenser, was used. This type of apparatus favors the production of spectra requiring only moderate excitation, as has been found in work with other elements. That such is the case was shown in these experiments with iodine by the great intensity of the band spectrum in the visible region as observed through the outer end of the discharge tube. Furthermore, three of the lines previously known to be excited strongly by active nitrogen and in a carbon arc,[3] were among the strongest of the lines

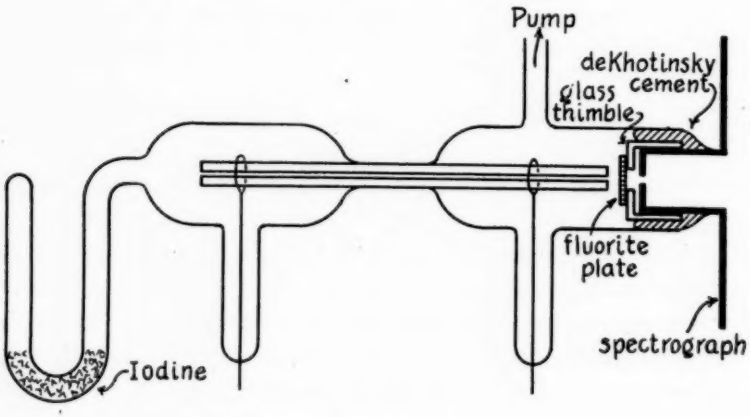


Fig. 1. The discharge tube.

obtained with the continuous current. Consequently, the spectra produced with this excitation have been designated as arc spectra. Different spectra were observed when an ordinary condensed spark discharge was used.

Great difficulty was experienced with the frequent appearance, on the fluorite window, of thin films which were opaque to the light of short wave-lengths. The presence of a small amount of air or water vapor would cause the rapid appearance of such a film. The deKhotinsky cement, used to fasten the discharge tube to the spectrograph, was acted on by the halogen vapors and may have been a further source of impurities which caused these films. The region of the tube surrounding the positive electrode quickly became coated with a dark deposit but no such deposit formed around the negative electrode. Accordingly the

electrode nearest the window was always made negative. It was found that the window would remain clean if the discharge tube were connected to the pump continuously through a liquid air trap. It was expected that these various impurities would give spectrum lines of their own but that such lines would be found in both the iodine and bromine spectra.

The standards of measurement used were lines of the many-line spectrum of hydrogen measured by Lyman[5] and the iodine line at 1876.40 measured by R. S. Mulliken and the author.[3] The wave-lengths of these hydrogen lines were originally estimated to be correct to within 0.3 A.U. but subsequent work has shown them to be considerably more accurate. The wave-lengths of the lines of the halogens should be accurate to 0.2 A.U.

## Iodine

For obtaining the iodine spectra an appendix containing ordinary resublimed iodine was attached to one end of the discharge tube. It was kept in melting ice. The other end of the discharge tube was continuously connected to the pump through a liquid air trap. There was, therefore, a pressure gradient in the iodine vapor through the system, the maximum pressure being 0.03 mm. The flow of the iodine vapor tended to sweep out impurities. When the discharge tube was closed off from the liquid air trap the CO bands came out strongly and some of the strongest iodine lines were tremendously reduced in intensity. Others of the strong lines were apparently unaffected. The lines which show this reduction markedly are indicated in the list of lines of Table 1. This effect did not occur when the discharge was passed through a mixture of hydrogen and iodine. A few spectra were obtained with a different discharge tube having large external tinfoil electrodes, excited by the transformer alone. The spectra thus obtained were practically the same as those given by the continuous current discharge but of lower intensity. One exposure was obtained by passing a condensed spark discharge from a small transformer through the tube with the internal electrodes. In this exposure the fluorite window was quickly coated over but several strong new lines appeared on the plate. None of the most intense arc lines was present, however. The three lines marked with an asterisk in Table 1 were present in this spark exposure with moderate intensity, much less than that of several other new lines, however. No attempt was made to measure the wave-lengths of the spark lines because of the lack of a comparison spectrum for fixing the position of these lines with respect to the standards. Table 1 gives

[5] T. Lyman, Memoirs Am. Acad. Arts and Sci., **XIII, III** (1906). Also "The Spectroscopy of the Extreme Ultraviolet."

TABLE I

*Iodine*

| Intensity | Wave-length | Wave-number | Intensity | Wave-length | Wave-number |
|---|---|---|---|---|---|
| 10 | †2062.1 | 48494 | 5 | 1390.9 | 71897 |
| 7 | 1876.4 | 53293 | 3 | 1383.4 | 72286 |
| 9 | 1844.5 | 54216 | 0 | 1382.3 | 72342 |
| 10 | +1830.4 | 54632 | 3 | 1368.3 | 73084 |
| 7 | 1799.2 | 55581 | 2 | 1367.7 | 73114 |
| 00 | 1789.6 | 55879 | 0 | 1366.6 | 73173 |
| 9 | +1782.9 | 56089 | 5 | 1361.2 | 73465 |
| 8 | 1702.3 | 58744 | 3 | 1358.1 | 73631 |
| 1 | 1675.4 | 59690 | 6 | 1355.5 | 73775 |
| 7 | +1642.5 | 60885 | 00 | 1354.3 | 73841 |
| 7 | 1641.1 | 60934 | 00 | 1352.4 | 73942 |
| 1 | 1639.2 | 61005 | 1 | 1350.3 | 74055 |
| 6 | +1617.9 | 61809 | 1 | 1349.0 | 74128 |
| 6 | 1593.8 | 62742 | 1 | 1343.7 | 74420 |
| 1 | 1582.8 | 63179 | 2 | 1340.9 | 74579 |
| 1 | 1545.9 | 64685 | 0 | 1339.9 | 74631 |
| 4 | +1526.6 | 65505 | 6 | 1336.7 | 74810 |
| 7 | 1518.3 | 65863 | 2 | 1330.4 | 75167 |
| 9 | 1514.8 | 66014 | 3 | (1317.7) | 75893 |
| 3 | 1507.3 | 66342 | 1 | 1314.1 | 76099 |
| 5 | 1493.2 | 66971 | 0 | 1313.6 | 76124 |
| 1 | 1486.1 | 67289 | 1 | 1303.2 | 76735 |
| 00 | 1466.7 | 68179 | 2 | *1300.6 | 76888 |
| 5 | 1466.0 | 68210 | 0 | 1296.6 | 77127 |
| 6 | 1459.2 | 68528 | 1 | 1291.4 | 77437 |
| 6 | +1458.2 | 68579 | 1 | 1289.6 | 77541 |
| 4 | 1457.6 | 68606 | 2 | *(1286.3) | 77742 |
| 4 | 1453.4 | 68806 | 0 | 1277.4 | 78281 |
| 5 | 1446.5 | 69131 | 2 | *1275.7 | 78386 |
| 0 | 1429.7 | 69947 | 2 | (1259.4) | 79400 |
| 8 | 1425.7 | 70143 | 3 | 1234.2 | 81024 |
| 4 | 1421.6 | 70345 | | | |
| 3 | 1400.2 | 71421 | | | |
| 00 | 1395.0 | 71685 | | | |
| 2 | 1393.3 | 71769 | | | |

† This line lies outside of the region here investigated but previous work shows it to belong to this group. Its intensity is therefore uncertain, but is certainly much greater than that of 1876.4

+ Greatly reduced in intensity when the CO bands appear.

( ) Coincide with lines obtained with bromine.

* Possibly spark lines.

the estimated photographic intensities, the wave-lengths, and the wave-numbers of the arc lines.

## BROMINE

Pure Kahlbaum bromine was placed in a small bulb which was connected to the discharge tube through a very fine capillary tube. As before, the discharge tube was connected continuously to the pump through a

liquid air trap. The proper size of capillary to reduce the bromine pressure to the right amount was found by trial. The pressure was further adjusted by altering the temperature of the bulb and capillary so that a current of 10 milliamperes would flow through the tube when the conditions of excitation were the same as with the iodine. No attempt was made to

TABLE II

*Bromine*

| Intensity | Wave-length | Wave-number | Intensity | Wave-length | Wave-number |
|---|---|---|---|---|---|
| 10 | 1633.6 | 61214 | 0 | 1400.8 | 71387 |
| 8 | 1582.4 | 63194 | 0 | 1397.5 | 71558 |
| 6 | 1576.5 | 63432 | 0 | 1394.5 | 71710 |
| 9 | 1575.0 | 63492 | 0 | 1391.3 | 71875 |
| 6 | 1540.8 | 64902 | 8 | 1384.6 | 72224 |
| 7 | 1531.9 | 65277 | 1 | [1379.6] | 72485 |
| 8 | 1488.6 | 67176 | 0 | [1363.5] | 73341 |
| 0 | 1484.0 | 67383 | 0 | [1351.7] | 73981 |
| 2 | 1477.1 | 67699 | 1 | [1347.3] | 74224 |
| 0 | 1468.2 | 68111 | 0 | [1335.6] | 74869 |
| 0 | 1466.4 | 68192 | 6 | (1317.8) | 75884 |
| 4 | 1463.4 | 68336 | 5 | 1316.9 | 75937 |
| 0 | 1459.9 | 68498 | 5 | 1310.0 | 76335 |
| 1 | 1457.0 | 68633 | 1 | (1286.4) | 77735 |
| 3 | 1449.9 | 68968 | 2 | 1279.7 | 78145 |
| 0 | 1442.3 | 69335 | 1 | 1266.3 | 78969 |
| 0 | 1435.8 | 69646 | 1 | 1261.8 | 79253 |
| 2 | 1423.0 | 70276 | 2 | (1259.4) | 79406 |
| 2 | 1416.5 | 70597 | | | |
| 1 | 1411.5 | 70847 | 1 | 1255.9 | 79621 |
| 1 | 1410.1 | 70915 | 4 | 1251.8 | 79886 |
| 0 | 1407.1 | 71070 | 0 | 1249.7 | 80017 |
| 0 | 1405.1 | 71167 | 2 | 1244.0 | 80384 |
| 0 | 1403.6 | 71243 | 5 | 1232.5 | 81137 |

( ) Coincide with lines obtained with iodine.
[ ] Coincide with lines obtained with chlorine.

measure the bromine pressure in the tube. If it were changed much either way the discharge would not pass. One spectrum was obtained with the condensed spark discharge. As with iodine, the strong lines of the spark spectrum were new ones. The strong arc lines appeared faintly. Table 2 gives the intensities, wave-lengths, and wave-numbers for the arc lines.

## CHLORINE

It was not convenient to prepare and handle pure chlorine gas so the chlorine spectrum was obtained by the use of compounds. One set of exposures was made to a continuous current discharge in hydrogen chloride gas. The hydrogen chloride was generated by warming a mixture of potassium acid sulphate and sodium chloride, and was dried by passing

it over phosphorous pentoxide. Its pressure was governed by the amount of warming and the adjustment of a stopcock. As with bromine the pressure was adjusted until it gave a discharge which behaved satisfactorily. The spectrum consisted of the many-line spectrum of hydrogen and seven strong lines. These same seven strong lines were the only prominent feature of a spectrum obtained from carbon tetrachloride which was used with a bulb and capillary in the same way as bromine. Consequently, the seven lines are ascribed to chlorine, the common element. Table 3 gives the data for these lines. Two of them agree within the limits of error with the two lines of wave-lengths 1351.6 and 1347.2 A.U. found by Millikan,[6] along with many other lines, in the hot-spark spectrum of chlorine.

TABLE III

*Chlorine*

| Intensity | Wave-length | Wave-number |
|---|---|---|
| 3 | 1396.5 | 71607 |
| 4 | 1389.9 | 71946 |
| 5 | (1379.6) | 72484 |
| 5 | (1363.5) | 73342 |
| 3 | (1351.7) | 73979 |
| 5 | (1347.2) | 74225 |
| 2 | (1335.8) | 74863 |

( ) Coincide with lines obtained with bromine.

It is of interest to note that there was no trace of a band spectrum which could be ascribed to the HCl molecule, although the excitation was probably most favorable for its production, if there is such a spectrum. Barker and Duffendack[7] found no band spectrum of HCl between 6000 and 2000 A.U. The present work places the lower limits at 1230 A.U. Apparently there is no electron transition in the HCl molecule of energy value between 2 and 10 volts. The non-existence of this band spectrum is in accord with Mulliken's[8] ideas concerning the instability of energized polar molecules.

## IMPURITIES

It is not certain that all of the lines listed in the above tables actually belong to the elements to which they are ascribed because some of them may have been emitted by the impurities unavoidably present. Since the substances used were quite pure they must have resulted principally from contamination from the electrodes and the deKhotinsky cement. Any lines emitted by such impurities should appear in the spectra of all three elements. Three of the less intense lines which are common to the

[6] R. A. Millikan, Phys. Rev. **23**, 1 (1924).
[7] E. F. Barker and O. S. Duffendack, Phys. Rev. **26**, 339 (1925).
[8] R. S. Mulliken, Phys. Rev. **26**, 29 (1925).

iodine and bromine spectra may be of this sort. The exposures made with the chlorine compounds were shorter so that these lines might not have shown on the plate even if present. The longest exposures were made with iodine so it may be that some of the weakest lines in its spectrum are those of such impurities but that they are not found in the bromine spectra because the exposures were too short to bring them out. Carbon and oxygen are probable impurities but their principal arc lines are not present. Five of the strong chlorine lines appear faintly in the bromine spectrum with the same relative intensities, indicating that the bromine used contained a small amount of chlorine. It is impossible, of course, to eliminate lines produced by an impurity present with only one of the halogens.

## Discussion

The most striking characteristic of these spectra is the progression of the long wave-length limits of the spectra towards longer wave-lengths with increasing atomic number. The minimum possible values of the radiating potentials, corresponding to the lines of longest wave-length, are thus fixed at 8.9, 7.6, and 6.0 volts for chlorine, bromine, and iodine, respectively. There are no indications of the convergence of iodine lines to a limit near 1400 A.U., the value corresponding to Smyth and Compton's[9] determination of the ionization potential of atomic iodine.

It was established by Paschen that the lines of the neon spectrum converge to two limits differing by 782 wave number units and Grotrian[10] showed the identity of these limits with the L x-ray absorption limits of the relativity doublet. Apparently the $Ne^+$ ion can exist in two different states differing in energy by an amount corresponding to the 782 wave number units. One would expect the neutral fluorine atom, having the same electron structure, to exist in two such states. The same should apply to the other halogen atoms which all presumably have similar outer structures. This is in accord with Pauli's[11] conclusion that the normal state of an atom lacking one electron from a completed group has the same multiplicity and inner quantum numbers as an atom having one extra electron of the same azimuthal quantum number. The normal state of these atoms should thus be a doublet $p$ state. If it is double (or, to put it another way, if there is an excited state close to the normal one) the energy difference between the two states should show itself as a frequently recurring doublet difference between lines of these spectra.

[9] H. D. Smyth and K. T. Compton, Phys. Rev. **16**, 501 (1920).
[10] W. Grotrian, Zeits. für Physik, **8**, 116 (1921).
[11] W. Pauli, Zeits. für Physik **31**, 765 (1925).

Each pair of lines corresponding to transitions from a given initial level to these two final levels should have this doublet difference.

The following differences, which are equal within the limits of the experimental error, are found between the wave numbers of the iodine lines. The estimated intensities are given in parentheses.

| | | | |
|---|---|---|---|
| (9) | 56089 | (10) 48494 | = 7595 |
| (7) | 60885 | (7) 53293 | = 7592 |
| (6) | 61809 | (9) 54216 | = 7593 |
| (1) | 63179 | (8) 55881 | = 7596 |
| (3) | 66342 | (8) 58744 | = 7598 |
| (6) | 68528 | (7) 60934 | = 7594 |
| (4) | 70345 | (6) 62742 | = 7603 |
| (3) | 72286 | (1) 64685 | = 7601 |
| (5) | 73465 | (7) 65863 | = 7602 |
| (00) | 73942 | (3) 66342 | = 7600 |
| (2) | 74579 | (5) 66971 | = 7608 |
| (0) | 76124 | (6) 68528 | = 7596 |

The fact that two of the lines appear twice indicates that some of these differences are the same purely by chance. On the other hand, the inclusion of so many of the strong lines is a point in favor of the reality and significance of this difference. Other recurring differences have been found but no other which groups as many lines or includes nearly as many of the strong lines of the spectrum.

Two such differences are found with the bromine lines. The wave numbers are as follows.

| | | | |
|---|---|---|---|
| (9) | 63492 | (10) 61214 | = 2278 |
| (10) | 67176 | (6) 64902 | = 2274 |
| (0) | 69646 | (0) 67383 | = 2273 |
| (1) | 70915 | (1) 68633 | = 2282 |
| (0) | 71243 | (2) 68968 | = 2275 |
| (1) | 80017 | (2) 77735 | = 2282 |
| (6) | 64902 | (10) 61214 | = 3688 |
| (8) | 67176 | (9) 63492 | = 3684 |
| (3) | 68968 | (7) 65277 | = 3691 |
| (0) | 71387 | (2) 67699 | = 3688 |
| (0) | 71875 | (0) 68192 | = 3682 |
| (1) | 79621 | (5) 75937 | = 3684 |
| (1) | 80017 | (5) 76335 | = 3682 |

The second difference of 3685 units is more probably the significant one because it includes more of the strong lines.

Six of the chlorine lines can be grouped into three pairs as follows.

| | | | | |
|---|---|---|---|---|
| (5) | 72484 | − (3) 71607 | = | 877 |
| (5) | 74225 | − (5) 73342 | = | 883 |
| (2) | 74863 | − (3) 73979 | = | 884 |

More accurate wave-length determinations must be had before a final decision can be made as to the reality of these recurring differences. Nevertheless, it seems highly improbable that the results with iodine and chlorine, at least, can be a matter of chance. If these three differences of 881, 3685, and 7600 units are the differences between the energies of the two lowest states of the respective atoms the resonance lines are those of wave-lengths 1379.6, 1540.8 and 1782.9 A.U. and the radiating potentials 8.95, 8.01, and 6.92 volts for chlorine, bromine and iodine, respectively. The reasons for believing the resonance line of iodine to be other than the 2062.1 line have been discussed elsewhere.[12] The 2062.1 and 1782.9 lines are the pair of longest wave-length which have the doublet difference.

In a recent paper Dymond[13] has shown that experiments on the fluorescence of molecular iodine indicate that the iodine atom has an excited state 1.1 volts above the normal state. The frequency difference of 7600 wave number units found here corresponds to an energy of 0.94 volts. It agrees very well with a value of 0.9 volt calculated by Franck by extrapolation from the neon frequency difference, to which Dymond briefly refers. Dymond's value is in error by the same amount as the value for the heat of dissociation of molecular iodine of 1.4 volts, which he uses. The discrepancy may be due to an error in this quantity, which is a difficult one to determine accurately, although it seems rather large to be accounted for in this way.

Hopfield[14] has shown that the normal level of the oxygen atom and of the sulfur atom is triple and Pauli[11] has given theoretical reasons for expecting it to be quintuple. We should expect a halogen ion of similar electron structure to have that same multiplicity. If so, the arc spectra of these elements must have lines converging to either three or five different limits (just as the neon spectrum converges to two limits). The great complexity of these spectra is thus quite easily understandable.

It might also be noted that if the normal state of the halogen atom is double one would expect to find two electron affinity spectra for each element, with the doublet difference between them. The one of shorter wave-length would appear only under conditions where a considerable

[12] Louis A. Turner and K. T. Compton, Phys. Rev. **25,** 791 (1925).
[13] E. G. Dymond, Zeits. für Physik **34,** 553 (1925).
[14] J. J. Hopfield, Phys. Rev. **21,** 710 (1923); Nature **112,** 437 (1923).

fraction of the atoms was in the higher of the two low states. So far as I am aware, such a doubling has never been noted in connection with the spectra ascribed to the electron affinity process.

I am greatly indebted to Professor Lyman for allowing me to use his spectrograph, and to his laboratory assistant, Mr. H. W. Leighton, for great help in performing the experiments. I also wish to thank Professor Lyman and Professor F. A. Saunders for their interest in this work and their helpful suggestions.

This work was done at the end of my year as National Research Fellow at Harvard University. I wish to take this opportunity to express my indebtedness to the National Research Council for making this year possible, and to thank all the members of the Physics Department of Harvard University for their courtesy and helpfulness which made it most profitable and pleasant.

PALMER PHYSICAL LABORATORY,
PRINCETON UNIVERSITY.
December 12, 1925.

Note added with proof April 6, 1926:

These $p_1 - p_2$ differences of 881, 3685, and 7600 wave number units, expressed in volts, are 0.11, 0.45, and 0.94 volts. They are in good agreement with the values 0.1, 0.5, and 0.9 predicted by Franck,[15] by extrapolation from the $Ne^+$ doublet difference. They also agree fairly well with the values calculated by Franck,[15] and by Wolf,[16] from the wave-lengths of the edges of the continuous absorption regions and heats of dissociation of the halogen molecules. Granting the correctness of the theoretical explanation of these continuous bands given by Dymond, Franck, and Wolf, accurate determinations of their wave-lengths combined with these doublet differences will give the most accurate values for the heats of dissociation of these molecules. The heat of dissociation of the iodine molecule so determined from Dymond's measurement of the edge of the band is $2.47 - 0.94 = 1.53$ volts, or 35.3 kg cal. per gram mol.

[15] J. Franck. Address given before the Faraday Society. To appear in the Transactions of that society.

[16] K. L. Wolf, Zeits. f. Physik **35**, 490 (1926).

# THEORY OF CURRENT TRANSFERENCE AT THE CATHODE OF AN ARC

By J. Slepian

Abstract

The relatively low temperature of the cathode in the case of such metals as copper and mercury, suggests that thermionic emission from the cathode is not essential and that some other factor may be more important in determining the current carried to the cathode in such cases. It appears that if the gas next to the cathode is sufficiently hot, thermal ionization in accordance with Saha's equation may account for much of the current to the cathode. Calculation shows that an upper limit for the required temperature is somewhat in excess of 4000°K for Ca, and 6000°K for Cu.

## Introduction

ALTHOUGH it is generally accepted that a cathode hot enough for thermionic emission is a necessary condition for the existence of an arc, a number of observations have been made which cast serious doubt upon the universal necessity of this condition. By an arc, is here meant a self-maintaining discharge with a drop at the cathode very considerably less than the normal cathode drop for the particular cathode material and adjoining gas in question. Thus H. Stolt[1] found that by sufficiently rapid motion of an arc over its cathode, the cathode would give no evidence of having been subjected to a very high temperature, and the author[2] has shown that transition from a glow discharge to an arc may take place while the cathode is still cold.

The case for the necessity of a thermionically active cathode has been very ably presented in a notable paper by K. T. Compton[3] who concludes that the greater part of the current carried at the cathode is by electrons emitted thermionically. However, the most convincing examples which Compton gives are the carbon and tungsten arc, in which because of the refractory properties of the materials, the cathodes may reach such high temperatures that their thermionic emission is sufficient to account for most of the current. For more volatile materials, such as mercury or copper, the thermionic theory meets a very serious difficulty in that it calls for a cathode temperature far in excess of the boiling point of the cathode material, whereas Hagenbach and Langbein,[4] in the case of

[1] Stolt, Zeits. f. Physik **26**, 95 (1924).
[2] Slepian, Jour. of Franklin Inst. (Jan. 1926).
[3] K. T. Compton, Phys. Rev. **21**, 266 (1923).
[4] Hagenbach and Langbein, Arch. Gen. **461**, 329 (1918).

copper, iron, and nickel find by optical methods that the temperatures of the cathodes are actually very near their boiling points.

### Thermal Ionization in an Arc, and Theory of Cathode

In the paper by K. T. Compton referred to above, the theory is very convincingly presented that in the positive column of an arc, the ionization is maintained principally by the high temperature existing there. Using Saha's equation[5] Compton calculates the degree of ionization to be expected at 4000°K, and finds it sufficient to account for the observed conductivity in the positive column of the arc.

It appears now that this theory of thermal ionization in the arc may be used also to explain the passage of current to the cathode, so that it is no longer necessary for the cathode to have a large electron emission or to be heated to a higher temperature than its boiling point. In fact, the thermal ionization in the gas, with reasonable assumptions as to temperature, is such that the arc current may be carried to the cathode by thermally generated positive ions.

### Quantitative Test of Theory of Cathode

To be sound, this theory of the cathode must meet certain quantitative tests. First, the observed cathode drop must be sufficient to carry the necessary current density of positive ions to the cathode. Second, the product of cathode drop and current density must more than make up for the heat lost from the hot gas to the relatively cold cathode.

Langmuir[6] has shown that when a plane electrode in contact with an ionized gas is given a sufficiently negative potential to repel all electrons, the current density flowing to the electrode and carried entirely by positive ions will be independent of the potential and equal to the random positive ion current density in the gas. This random current density, I, is defined as the current corresponding to the number of positive ions per second which pass in one direction through a square centimeter of an imaginary plane in the body of the gas. According to Langmuir,

$$I = \tfrac{1}{4}\, n\, v\, e \qquad (1)$$

where $n$ is the number of positive ions per cm$^3$, $v$ is the average translational velocity of the thermal agitation of the ions, and $e$ is the electronic charge.

The theory of the cathode here being developed, then, clearly requires that $I$ given by (1) shall also be the current density at the cathode.

[5] Saha, Phil. Mag. **40**, 472 (1920).
[6] Langmuir, G. E. Rev. **27**, 449 (1924).

If $n_1$ is number of molecules per cm³ in a gas at temperature $T$, then

$$n = n_1 \cdot x/(1+x) \tag{2}$$

where $x$ is the fraction of molecules thermally ionized and is given by Saha's equation for atmospheric pressure,

$$\log \frac{x^2}{1-x^2} = -\frac{5050\, V_i}{T} + 2.5 \log T - 6.69 \tag{3}$$

$V_i$ being the ionizing potential of the gas in volts.

For the velocity of agitation $v$ of the ions, we have

$$\frac{1}{2}\frac{M}{N} v^2 = \frac{1}{2} kT\ , \tag{4}$$

$M$ being the molecular weight of the ion, $N$ the number of molecules in a gram molecule of substance, and $k$ Boltzman's constant. Also

$$n_1 = n_0 \cdot 273/T \tag{5}$$

where $n_0$ is the number of molecules per cm³ of a gas at 273°K. Substituting (2), (4) and (5) in (1), we obtain

$$I = \frac{273}{4} n_0 e \frac{x}{1+x} \sqrt{\frac{Nk}{MT}} \tag{6}$$

For calcium we have $M=40$, $V_1=6.01$, and for copper $M=63.5$ and $V_i=7.69$. Substituting these values in (3) and (6), and taking $n_0=2.70\times10^{19}$, $e=4.77\times10^{-10}$, $N=6.06\times10^{23}$, $k=1.37\times10^{-16}$ the curves of Fig. 1 have been calculated.

Now the current density at the cathode of an arc is of the order of 100 amperes per cm² at atmospheric pressure. Examining Fig. 1, we see that for calcium and copper the temperatures of the vapors need to be from 4000° to 5000°, and 6000° to 7000° respectively for the positive ion density to be sufficient to carry the current to the cathode.

These temperatures are rather higher than those determined experimentally, but two considerations will serve to make these figures acceptable. First of all, this high temperature needs to exist only in a thin layer of gas next to the cathode and will therefore be very difficult to detect experimentally. Secondly, Saha's equation, which is the basis of this calculation, was derived on the supposition of thermodynamic equilibrium. But an ionized gas in an electric field is not in thermodynamic equilibrium, since the electrons take energy from the electric field and maintain themselves with a greater energy of agitation than that

of the neutral molecules. Thus Langmuir[7] finds that in a mercury arc the electrons have a temperature of over 10,000°K in a gas of less than 400°K. For a gas in an intense electric field therefore, the temperature (of the neutral molecules) necessary for a given degree of ionization will be much less than that called for by Saha's equation.

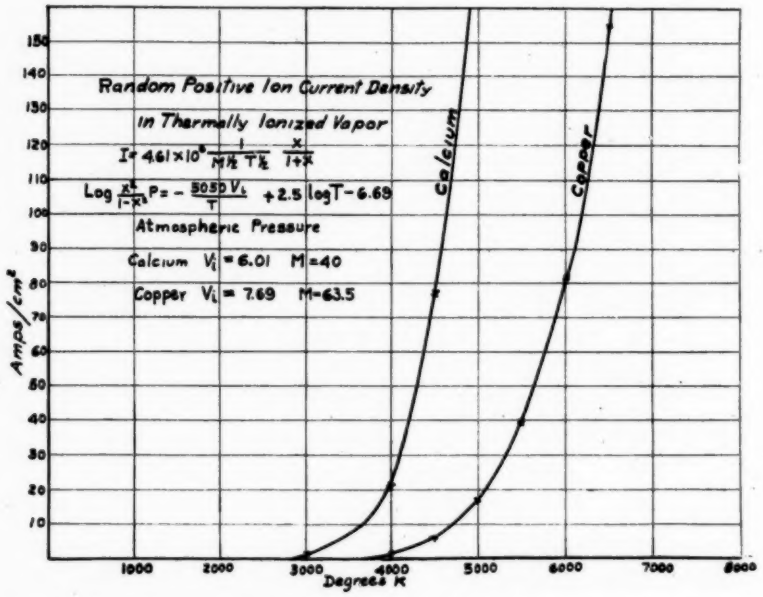


Fig. 1. Calculated variation of the temperature of the thermally ionized vapor with current density.

The electrode receiving the current of positive ions, as Langmuir has shown, is separated from the main body of ionized gas by a sheath containing only positive ions and neutral molecules, the electrons being repelled out of this sheath by the electric field. The thickness of this sheath varies with the applied potential, the applied potential being balanced by the space charge in the sheath. Langmuir gives the equation

$$I = \frac{2.336\times10^{-6}}{\sqrt{1834M}} \frac{1}{x^2} v^{3/2} \left(1+.0247\sqrt{\frac{T}{V}}\right) \qquad (7)$$

where $V$ is the applied potential in volts, $x$ the sheath thickness in cm, $I$ the current density in amperes/cm², $M$ the molecular weight of the ions, and $T$ the absolute temperature.

[7] Langmuir, G. E. Rev. **27**, 544 (1924).

If we take $I=100$, $V=15$, approximately the cathode drop for copper, $M=63.5$, and $T=6000°$, we find for the sheath thickness,

$$x=7.7\times10^{-4}\text{ cm}\,. \tag{8}$$

Within this sheath it is not necessary for the gas to be thermally ionized, but at the edge of the sheath the random positive ion current must have the full value of 100 amperes per cm². The vapor then must have a temperature of 6000° within $7.7\times10^{-4}$ cm from the cathode.

Since this separation from the cathode of $7.7\times10^{-4}$ cm is comparable with the mean free path in the gas at 6000°, the rate of heat flow across the space into the cathode can not be calculated from the thermal conductivity of the vapor in bulk, but an upper limit to this heat flow may be determined from the kinetic theory. If we assume that all the molecules which enter the positive ion sheath reach the cathode and there give up all the energy which they have in excess of that corresponding to the cathode temperature, the heat thus calculated as given up to the cathode will be in error by being too large, inasmuch as collisions with molecules of gas in the positive ion sheath, and partially elastic collisions with the cathode are not allowed for. Letting $n_1$ be the number of molecules per cm³ at temperature $T$ at the edge of the sheath remote from the cathode, the number of molecules which reach the cathode per second will be approximately

$$n_2=\tfrac{1}{4}n_1v_1 \tag{9}$$

$v_1$ being the mean translational velocity of the molecules, and since each molecule gives up the energy $3/2k(T-T_0)$, $T_0$ being the temperature of the cathode, we have as an upper limit for the heat lost to the cathode per second,

$$W=\tfrac{3}{8}n_1v_1k(T-T_0) \tag{10}$$

Letting $n_1=7.38\times10^{21}$, $v_1=1.14\times10^3T^{\frac{1}{2}}$ (copper), $k=1.37\times10^{-16}$, $T=6000°$, $T_0=2580°$ (boiling point of copper) we have in watts

$$W=1770\text{ watts} \tag{11}$$

If the cathode drop is 15 volts and the current density is 100 amperes per cm², the electrical input is 1500 watts. A large part, very likely the larger part, of this electrical input is communicated by bombarding positive ions directly to the cathode as heat, but not all of it, inasmuch as many of these positive ions will be reflected with a large part of their oncoming velocity, and many will collide with gas molecules in the space of $7.7\times10^{-4}$ cm of Eq. (8). Now remembering that (11) is much too high

for reasons given, we see that the electrical input is quite sufficient to make up for the heat lost to the cathode.

## Conclusion and Summary

The theory of thermal ionization in the electric arc, as advanced by Compton to explain the conductivity in the positive column, can also account for the passage of current to the cathode without requiring thermionic emission from the cathode. Calculations made for the copper arc, using equations of Langmuir and Saha, show that with a reasonable assumption of temperature of the vapor, the ionization is sufficient for the positive ions to carry the whole current to the cathode. The heat lost to the cathode by the hot vapor is of such an order that it can be made up for by the electrical input at the cathode.

A hot cathode is therefore not necessary for an arc. High temperature appears to be essential, but it may be in the gas immediately adjacent to the cathode, and need not be in the cathode itself.

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East Pittsburgh, Penn.
December 2, 1925.

# SECONDARY ELECTRONS FROM IRON

By H. E. Farnsworth

Abstract

**Secondary electron emission from iron as a function of energy of primary electrons, 0 to 300 volts.**—Curves showing ratio of secondary to primary electron current as a function of primary velocity were obtained with three forms of apparatus. Two have been previously described. The limiting curves obtained after red-heat treatment of samples of electrolytic Fe having very large crystals, and of chemically pure Fe obtained from Kahlbaum were similar to those obtained for a film of Fe deposited by evaporation. The curves all have sharp maxima at 1.2 and 7.0 volts with minima at 3.7 and 12.0 volts, and a less prominent maximum and minimum at 10.0 and 9.0 volts, respectively. Of several metals tried, Fe is the only one, in addition to Cu, for which such sharp maxima and minima have been obtained. Potentials corresponding to positions of the maxima are interpreted as critical potentials. A steady upward trend of the curve begins at 12.0 volts, which is interpreted as being due to ionized electrons. Slight changes in slope of the curve at higher potentials were observed but these varied with conditions, and hence are taken to have no real significance.

**Evidence of structure of evaporated Fe film.**—While previous results for Cu indicate that a layer deposited by evaporation is amorphous, the present results indicate that a similarly formed layer of Fe is crystalline.

## I. Introduction

A STUDY of secondary electrons produced when various metals, namely Ni, Cu, Ag, W, Pt, Pd, Mg, and Al, are bombarded by electrons varying in velocity from 0 to 250 volts has previously been described.[1,2] The study consisted first, in obtaining the number of secondary electrons as a function (1) of the primary velocity, (2) of the previous heat treatment of the metal, and (3), in the case of Cu, of the surface structure of the metal; second, in obtaining the velocity distribution of the secondary electrons for any given primary velocity.

In the case of all the metals investigated, a marked change in the magnitude of the secondary electron current, as well as in the shape of the secondary electron curve (curve showing secondary current as a function of primary velocity), was produced by heating the metals at red heat. The secondary current decreased with heat treatment for all metals tried, except Al and Mg for which it increased. A limiting curve for each metal was obtained after heating at red heat for some minutes. The changes in shapes of the secondary electron curves con-

[1] H. E. Farnsworth, Phys. Rev. **20**, 358 (1922).
[2] H. E. Farnsworth, Phys. Rev. **25**, 41 (1925).

sisted in the appearance of various breaks in the region of low velocities, 0 to 30 volts, but for no metal were these changes found to be so noticeable as they were for Cu. For this metal, four distinct maxima and four minima appeared in the region 0 to 30 volts, subsequent to red-heat treatment of the target. Further experiments on Cu showed that heat treatment produces two separate results: (1) outgassing of the metal, the rate of which varies with temperature, (2) removal of an amorphous surface layer, thus exposing the crystalline structure beneath, or the transformation of this amorphous layer into crystalline form. The second result occurred at a very critical temperature (dull red heat) in less than one minute. This change was detected by the sudden appearance of the several maxima and minima in the secondary electron curve, while previous heating at a slightly lower temperature for some time was only effective in lowering the curve without changing its shape—a result of outgassing.

Since the other metals were studied previous to the above observation, the above two effects were not separated, although they undoubtedly exist. No direct reason could be found as to why Cu is the only one of the investigated metals upon which heat-treatment produced such a marked effect, except that visible surface crystals of Cu are the most easily produced. Hence, it might be expected that other substances upon which prominent surface crystals are formed would show effects similar to Cu. To investigate this point the secondary electrons from Fe were studied. Fe was chosen because some very crystalline electrolytic Fe was available from Professor Terry.

The existence of several critical potentials, in the region 0 to 200 volts, at which the secondary electron current from Fe suddenly increases has been reported by Stuhlman.[3] These have been attributed to the photoelectric action of soft x-rays that are caused by electronic bombardment of the metal surface. More recently Petry[4] has obtained many slight changes in slope of the secondary electron curves for Fe, Ni, and Mo in the region zero to several hundred volts which, in the case of the higher potentials, do not appear to be due to the above cause. In the present experiments on Fe, special effort was made to obtain evidence of the existence of such critical potentials by taking observations at smaller voltage intervals than previously in the higher velocity region.

As reported in a previous article,[2] attempts were made to obtain velocity distribution curves of secondary electrons from Cu which would indicate the existence of inelastic collisions at critical voltages corre-

[3] Stuhlman, Science **56**, 344 (1922); Phys. Rev. **25**, 234 (1925).
[4] Petry, Phys. Rev. **26**, 346 (1925).

sponding to the position of the maxima in the secondary electron curves. Although some indication of such inelastic collisions was found, the results were not entirely satisfactory, so further investigation is described in the present paper.

## II. Apparatus and Procedure

As formerly described, two different forms of apparatus were used for study of Cu. The first, which was suitable for measuring only the ratio of the secondary to the primary current as a function of the primary velocity, consisted of a series of insulated molybdenum cylinders and diaphrams. The target to be studied was alternately interposed and withdrawn in front of a Faraday cylinder (Fig. 2, loc. cit.[2]). The second, which was designed for the purpose of obtaining an accurate measure of the velocity distribution of secondary electrons for any given primary velocity, consisted essentially of a conducting sphere at whose center the small target to be studied could be placed (Fig. 3, loc. cit.[2]). A nearly radial retarding field for secondary electrons could be applied between sphere and target so as to effect an accurate measure of the velocity distribution of secondary electrons. Although this apparatus was intended only for observations on velocity distribution of secondary electrons, it was found that results on the ratio of secondary to primary current for Cu agreed with those of a previously used apparatus, so that this apparatus was used to extend the investigation to Fe. As previously mentioned, it was necessary to keep the potential of the cylinder *C* (Fig. 3 loc. cit.[2]), which limits the primary beam, about 3 volts positive with respect to the sphere in order to keep the primary beam from spreading before reaching the target. While studying an Fe target it was found impossible to give *C* a potential for low primary velocities, which would prevent spreading of the primary beam. From the nature of the measurements, a spreading of the primary beam, so that some of it misses the target and strikes the sphere, results in an effective increase in the secondary current which cannot be separated from the true secondary current. Hence, the curve obtained was much higher than the correct one in the region of low velocities, the height varying with the potential of *C*. This effect was negligible for primary velocities higher than about 10 volts. In order to obtain the correct curve in the region below 10 volts, the apparatus shown in Fig. 2 (loc. cit.[2]) was used. By means of this apparatus the effect of scattering could be eliminated. It was at first thought that this scattering was due to the magnetic property of the iron target, but investigation indicated that it was independent of the position of the target and occurred very near the opening in the sphere

where the primary electrons enter, and that all of those electrons which are scattered suffer a very great change in direction, there being no electrons which are scattered through only a few degrees. These scattered electrons are probably the ones which pass close to the edges of the diaphram.

Since this scattering would change with the retarding potential of the sphere, it is obvious that it would seriously affect the results on velocity distribution of secondary electrons. In an attempt to eliminate this difficulty, the apparatus was modified as shown in Fig. 1 of the present paper. It is the same as that described on p. 46–47 of the above reference,[2] except for the addition of a Faraday cylinder $E$ surrounded by a co-axial cylindrical metallic shield $G$, both constructed of copper. The position of the cylinders is as shown, with the forward ends at the center of the

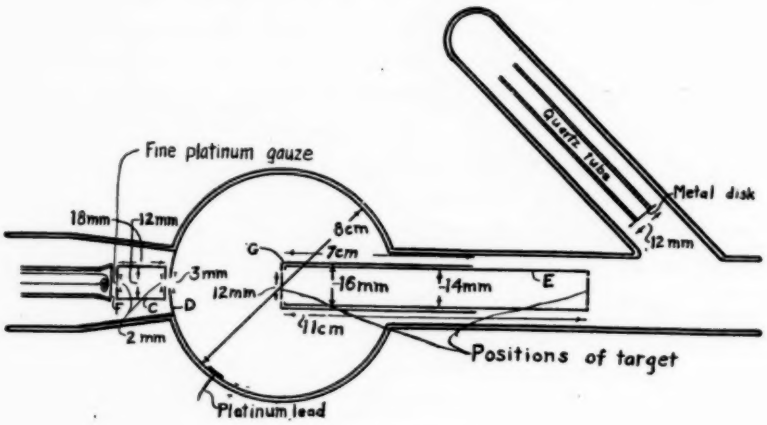



Fig. 1. Apparatus.

sphere. The shield was held in place and also insulated from the Faraday cylinder by three quartz rods of 1 mm diameter and 7 cm length. The rods were placed symmetrically and parallel to the axes of the cylinders and were held in place by small projections on the inner cylinder. The difference in diameters of the cylinders was just sufficient to insure a snug fit after inserting the quartz rods. The Faraday cylinder was fastened to and held in position by two glass rods (not shown in Fig. 1) upon which the target-mounting slid, so that the target could be moved along the axis of the cylinders, the plane of the target being perpendicular to the axis. Circular openings in the ends of the cylinders were just large enough to permit the free passage of the target. The target could thus be placed in the forward position, flush with the end of the cylindrical shield, or withdrawn (by means of a magnetic control) into the side-tube

where it could be heated at red heat by high-frequency induction. The Faraday cylinder and target were kept connected at all times. The presence of the shield prevented any electrons from reaching the Faraday cylinder by multiple reflection from the sphere or by scattering.

The procedure was then to measure (1) the current to the target when in the forward position, (2) the total or primary current to the Faraday cylinder and target with the latter at the back end of the cylinder, this being a measure of the total current striking the target since no electrons could escape from the cylinder.[5] Subtracting the first current from the second then gives the secondary current. The ratio of this secondary current to the primary current, as a function of the retarding potential on the sphere, should then give a measure of the velocity distribution of secondary electrons for any given primary velocity. The secondary electron curve can also be obtained by taking the ratio of secondary to primary current as a function of primary velocity, the potential of the sphere being constant and equal to that of the target. This procedure differs from that used with apparatus shown in Fig. 3 of above reference,[2] in the method of measuring the total current. Any scattering of the primary electrons on entering the sphere S will not influence the measurement of the ratio of secondary to primary current provided that this scattering remains the same for these two observations at any particular primary velocity. Thus the results should not be affected by this change even if the scattering changes with the primary velocity or with the potential of the sphere.

Conditions previously described regarding vacuum were obtained in the present experiments. In addition a charcoal tube was attached to the present apparatus and immersed in liquid air. This, however, did not change the results. Helmholtz coils were used to compensate the earth's magnetic field.

## III. Results

*A. Results on ratio of secondary to primary current, as a function of primary velocity*, for Fe are shown in Fig. 2. Curve 1 is the limiting curve for an Fe target obtained after red-heat treatment. The form of the curve in the lower region, below about 13 volts, was found to be the same for two specimens of Fe—a very crystalline electrolytic Fe and a chemically pure Fe obtaiued from Kahlbaum—after red-heat treatment. It was also obtained with three different forms of apparatus: that shown in Fig. 1 of the present paper, and those of

[5] Measurements of current to the cylinder and target for various positions of the target showed that the cylinder was longer than necessary to be completely absorbing.

Figs. 2 and 3 of above reference.[2] The agreement was in all cases very close except in the one referred to above which was affected by scattering.

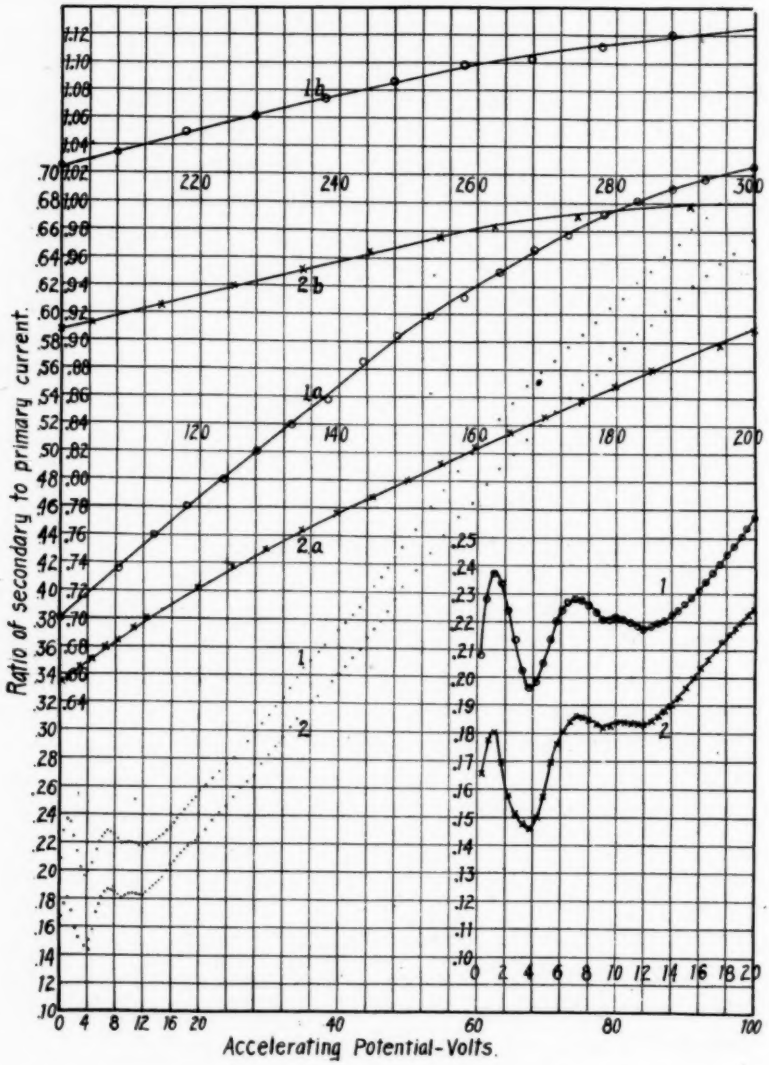



Fig. 2. Secondary electron curves. 1 is for Fe after red-heat treatment. 2 is for a film of Fe deposited by evaporation. 1a and 2a are extensions of 1 and 2, respectively, from 100 to 200 volts and are plotted to the upper scale of ordinates. 1b and 2b are extensions of 1a and 2a, respectively, from 200 to 300 volts. The inset is a magnified portion of curves 1 and 2.

It is seen that there are sharp maxima at 1.2 and 7.0 volts with minima at 3.7 and 12.0 volts and a less prominent maximum and minimum at

10.0 and 9.0 volts,[6] respectively, with a steady upward trend beginning at 12 volts. In the region above about 13 volts there are several slight changes in slope noticeable. Although the general trend of the curve in this region was similar in all of the above mentioned cases, the positions of the slight changes in slope were found to vary with uncontrollable conditions as well as with the form of apparatus and the particular target used. It may be mentioned, however, that of the curves obtained, it is possible to select a large number which agree fairly well with curve 1 in the positions of these slight changes in slope but this procedure has no justification. The disagreement in the position of these slight breaks does not appear to be due to errors in observation since it was possible to accurately repeat a set of observations within a short time after they were taken.

The effect of outgassing was to decrease the secondary current as in the case of most of the other metals previously investigated. The curve obtained subsequent to baking at 400°C but previous to red-heat treatment was a smooth curve without maxima and minima and higher than curve 1.

Curve 2 is for a film of Fe deposited by evaporation and is seen to be lower than curve 1 but agrees with it in the position of the maxima and minima.

Correction has been made in these curves for contact potential between source and target and for change in potential of target due to heating, as described in a previous paper (loc. cit.[2]).

B. *Velocity distribution of secondary electrons from Fe.* Although the apparatus shown in Fig. 1 was designed to give a more accurate measure of velocity distribution than that previously obtained, an unanticipated difficulty was encountered when attempting to effect these measurements. The number of electrons entering the Faraday cylinder, with the target withdrawn to the far end, was found to be equal to the number which struck the target when in the forward position, only when the potential of the sphere was equal to that of the cylinder and target. As the potential of the sphere was made negative with respect to the target and cylinder, the number of electrons which entered the Faraday cylinder, with the target back, became less than the number striking the target when in the forward position, the difference going to the shield instead of to the Faraday cylinder. This spreading of the primary beam at the opening of the Faraday cylinder was obviously caused by the distorted

[6] The maximum at 10 volts was in many cases not as noticeable as in the curve shown. In some curves there was no actual maximum but a region of the curve separated from the rest by changes in slope at 9.0 and 12.0 volts.

field which resulted when the target was withdrawn. This distortion increased as the retarding potential on the sphere was increased so that the condition necessary for the measurement of velocity distribution of secondary electrons effected circumstances which made this measurement impossible. Preliminary measurements with the apparatus before the Faraday cylinder was inserted were sufficient to show that, in general, the velocity distribution of secondary electrons from Fe is similar to that of other metals, i.e. the number of low-velocity secondary electrons increases as the primary velocity increases but these results are not accurate enough to decide the existence or non-existence of the effect in question.

## IV. Discussion

The fact that the same type of secondary electron curve was obtained for two different specimens of Fe and with three different forms of apparatus is sufficient evidence of the validity of this curve. The occurrence of sharp maxima and minima at certain critical potentials in the low-velocity region of the above curve for Fe shows the existence of a phenomenon which has an effect on the secondary electron curve as noticeable as that observed in the case of Cu.

In the previous report on Cu it was shown that if one makes certain simple assumptions regarding inelastic collisions a curve containing maxima and minima is obtained which approximates closely to the experimental curve, at least in the region of lower velocities. The maxima occur at certain critical potentials followed by minima. If the work function of the metal is taken into account, as noted by Petry,[4] all primary electrons strike with energy due to the accelerating potential plus that acquired in falling through the surface and all secondary electrons with energy less than that corresponding to the work function will be unable to escape.[8] Hence an interval equal to at least the work function in volts might be expected to separate a maximum from the minimum following it. This interval, however, would depend on the type of critical potential. For an ionization potential it would depend on the distribution of the residual energy between the two colliding electrons after impact. If the energies of the two electrons are equal, the interval referred to above should be twice the work function. If either of the electrons has all of the residual energy after impact, the interval should be equal to the work function. In the case of a resonance posential it would also be equal to

[8] From this point of view it is evident that the effect of critical potentials of solids having a value smaller than the work function cannot be observed at primary velocities corresponding to the critical potential, since the slowest primary electron has an energy equal to the work function.

the work function. Referring to the present results on Fe, the difference between the maximum at 7.0 volts and the minimum at 12.0 volts is the same as the work function ascribed to Fe by Thomas.[7] There is, however, a slight minimum and maximum between these two values. The difference between the first sharp maximum at 1.2 and minimum at 3.7 volts is seen to be only one-half of the above work function of Fe. It therefore seems necessary to conclude either that the work function of Fe is less than 5 volts or that some such condition as the following exists: that the energy loss at the first inelastic collision is only equivalent to 3.7 volts but the probability of its occurrence is negligible until an actual accelerating potential of 1.2+5.0 volts is reached. (Eldridge[9] has shown that a similar idea is necessary to explain the 6.7 resonance potential of Hg vapor.) The residual energy of 2.5 volts is not sufficient to allow the election to escape through the surface so that the secondary current begins to decrease at this point. When the primary electron has energy of 3.7+5.0 volts, after falling through the surface layer, it loses 3.7 volts on collision and has 5.0 volts left which will just allow it to emerge through the surface. Thus the secondary current begins to increase at this point. A knowledge of the velocity distribution of secondary electrons should throw light on this question. At any rate, the gradual rise in the curve beginning at 12.0 volts is undoubtedly due to electrons resulting from ionization. It may be noted that Petry's[4] curve for Fe shows a maximum at about 7 volts with a minimum at about 10 volts.

In the study of Cu it was found that excessive heating of the target near the melting-point resulted in a smoothing out of the various maxima and minima, which was attributed to a change in crystal structure. No such effect was observed for Fe, which would indicate that the crystal structure of Fe is more stable than that of Cu.

The curve for evaporated Fe (curve 2, Fig. 2) is seen to be similar in form to curve 1, at least in the region of low velocities. Previous results show that depositing an evaporated Cu film onto a Cu target causes the disappearance of the sharp maxima and minima—a result which suggests that evaporated Cu is amorphous. The similarity of curves 1 and 2 for Fe then indicates that evaporated Fe is not amorphous but crystalline, the difference in absolute values indicating that evaporated Fe is more gas-free than the other.

Although the present results prove the existence of critical potentials for Fe in the region below about 13 volts accelerating potential, there appears to be no true critical potentials for higher accelerating potentials.

[7] C. H. Thomas, Phys. Rev. **25**, 331 (1925).
[9] J. A. Eldridge, Phys. Rev. **20**, 459 (1922).

Since this article was first written, two papers by A. Becker[10] have appeared on velocity of secondary electrons from Pt covered with lampblack and from pure platinum. They, however, contain nothing immediately related to the present results.

These experiments were performed at the University of Wisconsin during the past summer and the writer expresses his appreciation to Professor C. E. Mendenhall for the privilege of continuing the investigation in that laboratory.

UNIVERSITY OF MAINE,
ORONO, MAINE.
November 10, 1925.

[10] A. Becker, Ann. d. Physik **78,** 228 (Nov. 1925); **78,** 253 (Nov. 1925).

# THE RESONANCE POTENTIALS OF HELIUM AND DOUBLE IMPACTS BY ELECTRONS IN HELIUM

By George Glockler*

Abstract

**First and second critical potentials in helium.**—A tube with five platinum electrodes was used to determine the first two critical potentials in carefully purified helium. The *inelastic impact method of Franck* was used over a pressure range from 0.13 to 13.00 mm and gave values of $19.87 \pm 0.06$ for the first, and $20.62 \pm 0.06$ volts for the second critical potential. The *method of Lenard* for pressures from 0.96 to 2.85 mm gave values of $19.83 \pm 0.02$ and $20.57 \pm 0.06$ volts. The spectroscopic value corresponding to the transition $1S \rightarrow 2s$ ($A$) is 19.77 volts and that corresponding to $1S \rightarrow 2S$ ($B$) is 20.55 volts. The observed difference between first and second critical potentials, 0.75 (Franck method), 0.74 volts (Lenard method), is in excellent agreement with the spectroscopic value of the difference $B-A$, 0.78 volts. The average *initial velocity correction* was determined by two different methods which led to concordant results.

**Critical potentials in helium due to double impact.**—Three critical potentials in helium near 40 volts were observed by the Franck method. The values are 39.62, 40.47 and $40.96 \pm 0.04$ volts, the last being the most prominent. The spectroscopic values for double impacts are 39.54 volts ($1S \rightarrow 2s$ and $1S \rightarrow 2s$), 40.32 volts ($1S \rightarrow 2s$ and $1S \rightarrow 2S$), and 40.89 volts ($1S \rightarrow 2s$ and $1S \rightarrow 2P$). The observed difference between the first critical potential and the most prominent double impact point is 21.09 volts, in excellent agreement with the spectroscopic value of the transition $1S \rightarrow 2P$, 21.12 volts.

SEVERAL experimental determinations of the critical potential of helium have been made by the method of electron impact.[1] Franck[2] has cleared up the situation regarding interpretation on the basis of Lyman's spectroscopic measurements[3] and at present it is accepted that the first critical potential of helium at 19.77 volts corresponds to a transition ($1S \rightarrow 2s$) from the normal to a metastable state.

In determining the critical potential of a gas by the method of electron impact a correction has to be made for the velocity distribution of the electrons, their initial velocity and for the contact potential difference between the filament and the other electrodes. In addition the efficiency of impact must be considered as shown by Sponer[4] and Dymond.[5] Franck

* National Research Fellow in Chemistry.

[1] K. T. Compton and F. L. Mohler, Bull. Nat. Res. Council, No. 48, p. 85 (1924).

[2] Franck, Zeits. f. Physik, **11**, 155 (1922).

[3] Lyman, Science, **56**, 167 (1922).

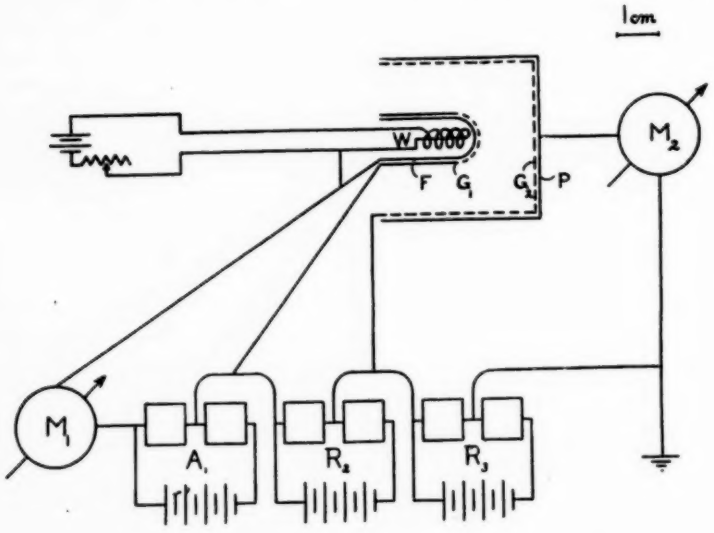
[4] Sponer, Zeits. f. Physik, **7**, 185 (1921).

[5] Dymond, Proc. Roy. Soc., **A107**, 291 (1925).

and Knipping[6] determined the last two corrections in the case of helium by measuring the volt-difference between the kinks in a current-potential curve corresponding to a single impact (type $A = 1S \rightarrow 2s$) and a double impact (type $A = 1S \rightarrow 2s$ and type $B = 1S \rightarrow 2S$). However, Dymond[5] has shown that in his experiments this difference is not 20.55 volts, but 20.9 volts, and he accordingly assumes that the double impact must be of the type $BB$. In this investigation the two single impacts appear as usual. The double impact that is most prominent occurs at 40.96 volts (corrected) and the difference between this point and the first drop is 21.09 volts corresponding in magnitude to the transition $1S \rightarrow 2P$. Indications were obtained at higher pressures which tend to show that the other types of double impact $AA$ and $AB$ also can occur.

## Experimental Procedure

The five-electrode tube used is shown in Fig. 1. All electrodes are of platinum. The gauzes $G_1$ and $G_2$ are 52 mesh/inch and the wire is 0.004 inch thick. $W$ is a tungsten filament that heated the calcium oxide




covered platinum thimble $F$ by radiation. $F$ is an equipotential surface. Between $F$ and gauze $G_1$ an accelerating field is applied by means of the potentiometer $A_1$ of 10,000 ohms. $M_1$ is a galvanometer that measures the total emission from $F$. This total current was kept constant by regulating the external resistance in the heating circuit. $M_2$ is a galvanometer of 50,000 megohms sensitivity (880 ohms resistance) and is used

[6] Franck and Knipping, Zeits. f. Physik, **1**, 320 (1920).

at a scale distance of 3 meters. Both galvanometers were read on the same scale by means of two telescopes.

The helium was purified by sparking with a little added oxygen for 24 hours and then it was passed through a quartz tube containing magnesium ribbon, copper shavings, and granular copper oxide at 550°C, thus removing nitrogen, oxygen, and hydrogen. It was further purified by passing through a trap containing cocoanut charcoal immersed in liquid air and stored over phosphorus pentoxide. It was admitted into the experimental tube by means of a mercury trap so that no stopcocks were used at any place in the vacuum system. The pressures were read on a McLeod gauge. The voltages on the potentiometer were measured by means of a standard cell before and after each run. Readings were taken in steps of 20 ohms which corresponds to about 0.1 volt, since the battery supplying the potentiometer was 50 volts. Liquid air was kept on a trap to remove mercury vapor, except in the experiments using the Lenard method.

## The Experimental Results

1. *The Franck method.* The current potential curves were obtained by two methods. The first of these was the inelastic impact method of Franck, in which the drop in electron current reaching the plate was determined as a function of the first accelerating field. No retarding

TABLE I

*Franck method*

| Run | Press. mm Hg | Total drop across potentiometer before | after | Peaks of difference curves First ohms | volts | Second ohms | volts | Third ohms | volts | Filament volts | amps. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.13 | 34.06 | 34.06 | ..... | ..... | .... | ..... | | | 4.30 | 4.40 |
| 1 | .81 | 34.06 | 34.06 | 6220 | 21.18 | 6410 | 21.82 | | | 5.00 | 5.40 |
| 2 | .81 | 34.06 | 34.06 | 6226 | 21.20 | 6410 | 21.82 | | | 4.90 | 5.30 |
| 3 | .83 | 34.06 | 34.06 | 6190 | 21.11 | .... | ..... | | | 4.80 | 5.25 |
| 4 | .83 | 34.06 | 34.06 | 6202 | 21.12 | 6450 | 21.97 | | | 5.00 | 5.70 |
| 5 | 3.35 | 34.06 | 34.06 | 6218 | 21.18 | .... | ..... | | | 5.25 | 5.75 |
| 6 | 0.75 | 50.72 | 50.72 | 4130 | 20.93 | 4310 | 21.86 | | | 4.80 | 5.30 |
| 7 | 2.63 | 50.72 | 50.72 | 4170 | 21.15 | .... | ..... | 8320 | 42.19 | 5.50 | 6.50 |
| 8 | 5.50 | 50.84 | 50.84 | 4166 | 21.18 | .... | ..... | 8310 | 42.25 | 5.80 | 7.00 |
| 9 | 5.50 | 50.72 | 50.71 | 4158 | 21.10 | .... | ..... | 8330 | 42.25 | 5.80 | 7.20 |
| 10 | 5.50 | 50.72 | 50.72 | 4164 | 21.12 | .... | ..... | 8330 | 42.25 | 5.70 | 6.70 |
| 11 | 1.01 | 50.78 | 50.66 | 4166 | 21.13 | 4350 | 22.06 | .... | ..... | 4.80 | 6.10 |
| 12 | 1.83 | 50.64 | 50.63 | 4186 | 21.19 | .... | ..... | 8350 | 42.28 | 5.00 | 5.80 |
| 13 | 2.91 | 51.32 | 51.29 | 4183 | 21.23 | .... | ..... | 8240 | 42.27 | 5.25 | 6.25 |
| 14 | 3.70 | 51.17 | 51.13 | 4142 | 21.20 | .... | ..... | 8260 | 42.25 | 5.50 | 6.50 |
| 15 | 5.00 | 51.11 | 51.10 | 4122 | 21.06 | .... | ..... | 8260 | 42.20 | 5.60 | 6.75 |
| 16 | 13.00 | 50.97 | 50.96 | 4134 | 21.07 | .... | ..... | 8270 | 42.15 | 6.20 | 7.70 |
| | | | Average values | | 21.14 | | 21.89 | | 42.23 | | |
| | | | Average deviation | | ±.06 | | ±.07 | | ±.04 | | |

field was applied between the second grid and the plate. The experimental curves obtained by this method are shown in Fig. 2 (*B* and *D*) in which galvanometer readings are plotted against potentiometer resistance.

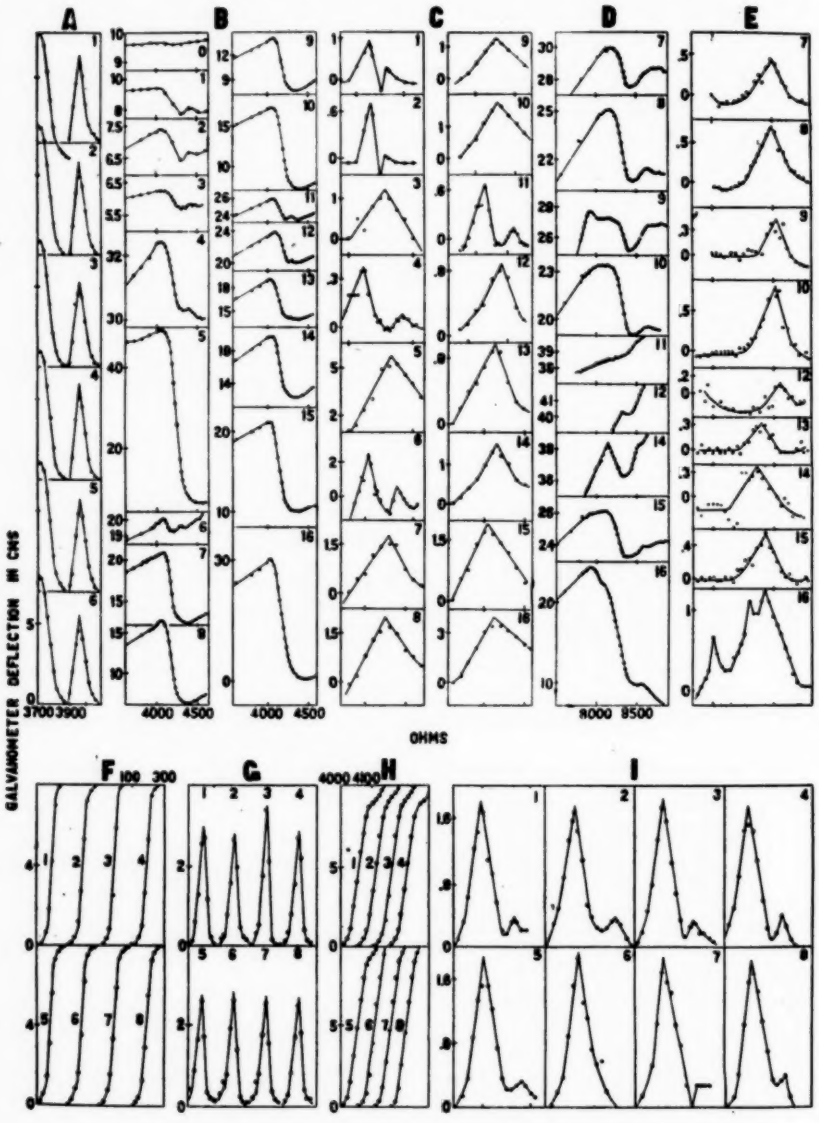



Fig. 2. Experimental current-potential curves.

These curves were then differentiated by taking the difference between successive galvanometer readings and replotting against potentiometer resistance. These difference-curves are shown in Fig. 2 (*C* and *E*). The

maxima of the difference-curves correspond to the points where *most* of the electrons from the filament have obtained enough energy to resonate. For the measurements in the neighborhood of 20 volts, the voltages corresponding to the maxima for the first current-drop are given in Table I, column 6, and for the second drop in Table I, column 8.

Measurements in the neighborhood of 40 volts were also made in the last eight experiments, and the peaks obtained are shown in Fig. 2 (*D* and *E*), and given in Table I, column 10.

*The initial velocity correction* (*Method I*). This correction was determined in a vacuum by applying a constant accelerating field ($A_1$) between $F$ and $G_1$ of about 20 volts and by noting the decrease in plate-current as the retarding field ($R_2$) between $G_1$ and $G_2$ was increased. The correction is negative due to a contact potential between $F$ and $G_1$.

If $P_c$ is this contact potential and $V_i$ is the actual velocity with which the electrons leave the filament, then a given electron can just reach the plate when

$$A_1+V_i+P_c=R_2 \tag{1}$$

where $A_1$ and $R_2$ are the accelerating and retarding fields respectively.

The results obtained are shown in Fig. 2 (*A*), where both the experimental curves and the difference curves are given, the latter being displaced to the right. In these curves galvanometer readings are plotted against the resistance, which gives the retarding field. It will be seen that the *first* drop in current due to $R_2$ occurs at 3700 ohms (=18.80 volts). This initial drop must be due to the slowest electrons. If they leave the filament with zero velocity, then

$$A_1+P_c=R_2 \tag{2}$$

and the contact potential is found to be acting as a retarding potential (i.e., $P_c=-1.6$ volts).

TABLE II

*Data for determination of initial velocity correction* (*pressure* $10^{-6}$*mm*Hg)

| Run | Total voltage first field $A_1$ before | Total voltage first field $A_1$ after | Total voltage second field $R_2$ before | Total voltage second field $R_2$ after | Field $A_1$ ohms | Field $A_1$ volts | Field $R_2$ at peak ohms | Field $R_2$ at peak volts | Correction $R_2-A_1$ | Filament amps | Filament volts |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 50.66 | 50.66 | 50.41 | 50.41 | 4000 | 20.26 | 3770 | 19.00 | −1.26 | 3.70 | 4.70 |
| 2 | 50.80 | 50.66 | 50.48 | 50.37 | " | 20.29 | " | 19.01 | −1.28 | 3.70 | 4.75 |
| 3 | 51.39 | 51.35 | 51.19 | 51.17 | " | 20.56 | " | 19.30 | −1.26 | 3.80 | 4.00 |
| 4 | 51.29 | 51.26 | 51.11 | 51.09 | " | 20.51 | " | 19.26 | −1.25 | 3.70 | 3.80 |
| 5 | 51.18 | 51.19 | 50.99 | 51.01 | " | 20.47 | 3760 | 19.18 | −1.29 | 3.75 | 3.85 |
| 6 | 50.98 | 50.97 | 50.80 | 50.79 | " | 20.38 | " | 19.10 | −1.28 | 3.75 | 3.90 |
| | | | | | Average correction | | | | −1.27 volts | | |

The velocity distribution was thus determined with no gas in the tube. It might be thought that the distribution is different with helium present. However, the critical points should then show some trend with pressure. No such trend was observed. Of course the presence of the gas necessitates a greater heating current in the tungsten filament $W$ as a good deal of heat is conducted away by the gas.

From the difference-curves the average correction was determined. The results are given in Table II.

2. *The Lenard method.* In the second method, Lenard curves were taken with a constant large retarding field (30 volts) between the second gauze $G_2$ and the plate. The results are shown in Fig. 2 ($H$ and $I$) and they are summarized in Table III.

TABLE III

*Lenard method*

| Run | Press. mm Hg | Total drop before | Total drop after | Peaks of diff.–curves First ohms | First volts | Second ohms | Second volts | Filament amp | Filament volts | Liq. air | Diff.–curve peaks for correction ohms | Diff.–curve peaks for correction volts |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.08 | 50.78 | 50.78 | 4102 | 20.82 | 4240 | 21.53 | 5.45 | 5.50 | off | 202 | 1.02 |
| 2 | 1.21 | 50.71 | 50.71 | 4112 | 20.85 | 4284 | 21.71 | 5.60 | 6.00 | " | 194 | 0.98 |
| 3 | 1.36 | 50.69 | 50.69 | 4114 | 20.85 | 4246 | 21.52 | 5.65 | 6.05 | " | 206 | 1.04 |
| 4 | 0.96 | 50.61 | 50.61 | 4108 | 20.79 | 4250 | 21.51 | 5.43 | 5.75 | " | 198 | 1.00 |
| 5 | 1.21 | 50.60 | 50.60 | 4118 | 20.84 | 4280 | 21.61 | 5.55 | 6.00 | " | 196 | 0.99 |
| 6 | 2.85 | 50.60 | 50.60 | 4118 | 20.84 | .... | ..... | 6.07 | 6.62 | " | 194 | .98 |
| 7 | 1.14 | 50.50 | 50.50 | 4122 | 20.82 | .... | ..... | 5.60 | 6.10 | on | 194 | .98 |
| 8 | 1.15 | 50.48 | 50.48 | 4124 | 20.82 | 4270 | 21.55 | 5.55 | 6.00 | " | 193 | .98 |
| | | Average values | | | 20.83 | | 21.57 | | | | | 1.00 |
| | | Average deviation | | | ± .02 | | ± .06 | | | | | ± .02 |

The total rise in positive current was about 9 cm on the galvanometer scale in runs Nos. 1 to 5. In run 6 the total rise was 17 cm. However, the two last runs were made with liquid air on the charcoal so as to remove mercury vapor and the total rise was only 3 cm, showing that most of the current was probably due to positive ions of mercury produced by the metastable helium atoms.

*The initial velocity correction* (*Method II*). Before the Lenard experiments could be made the filament had to be recoated with calcium oxide as its emission had decreased. The correction for the recoated filament was determined by Method I as described above and Method II as described below. Both gave identical results.

In Method II, instead of making separate experiments in a vacuum with two fields, the correction was now determined with gas in the tube at the same pressure as was to be used in the Lenard experiment immediately following. The increase in electron current to the plate was

measured as the first accelerating field was increased from 0.5 volt to 1.5 volts while the other fields were zero. After such an experiment the retarding field ($R_3$) between $G_2$ and the plate was applied and a Lenard experiment was performed.

The correction for the recoated filament amounted to $-1.00$ volt, while previously it was $-1.27$ volts. The difference is presumably due to a change in contact-potential. It is important then to measure the correction for each filament and to make sure that it does not change.

The results are shown in Fig. 2 ($F$ and $G$) and the last column of Table III. The figures given are the voltages $A_1$ which have to be applied in order to get the "average" electron to come through grid $G_1$ with approximately zero velocity and hence the correction has to be subtracted from the value of $A_1$ found in the later Lenard experiments. A few measurements of the correction were also made by Method I on the new filament and gave concordant results.

## Summary of Results

When the proper corrections as described above are applied to the critical points, the values given in Table IV are obtained.

Table IV

| Type | Transitions | Volts Spectroscopic | Franck method | Lenard method | Peak No. |
|---|---|---|---|---|---|
| $A$ | $1S\rightarrow 2s$ | 19.77 | $19.87\pm 0.06$ | $19.83\pm 0.02$ | 1 |
| $B$ | $1S\rightarrow 2S$ | 20.55 | $20.62\pm .06$ | $20.57\pm .06$ | 2 |
| $AA$ | $1S\rightarrow 2s$ and $1S\rightarrow 2s$ | 39.54 | 39.62 | ..... .... | 3 |
| $AB$ | $1S\rightarrow 2s$ and $1S\rightarrow 2S$ | 40.32 | 40.47 | ..... .... | 4 |
| $AC$ | $1S\rightarrow 2s$ and $1S\rightarrow 2P$ | 40.89 | $40.96\pm .04$ | ..... .... | 5 |
| $BB$ | $1S\rightarrow 2S$ and $1S\rightarrow 2S$ | 41.10 | ..... .... | ..... .... | |
| $BC$ | $1S\rightarrow 2S$ and $1S\rightarrow 2P$ | 41.67 | | | |

These values depend on the corrections applied. However, the differences between the peaks are independent of the initial velocity correction and the following quantities are obtained.

Table V

| Transition | Spectroscopic | Franck method | Lenard method |
|---|---|---|---|
| $B-A$ | 0.78 volts | 0.75 volts | 0.74 volts |
| $AC-A$ | 21.12 | 21.09 | .... |

## Discussion of Results

It is seen that at a pressure of 0.13 mm (Fig. 2 $B$, Run $O$) no drop in current was obtained. At this pressure the mean free path of the electron (0.96 cm) is nearly as great as the distance between the gauzes (1.1 cm). At a pressure of 1 mm the m.f.p. of the electron is 1.25 mm, while the distance between the filament and grid $G_1$ is 0.5 mm. In some of the

experiments the pressure was so high that some of the electrons must have made impacts in the region between filament and grid $G_1$. However, the voltages at which the maximum occurs do not seem to be different.

In the Franck method a small retarding field of 0.34 volt between the plate $P$ and grid $G_2$ was employed in runs 1 to 5, but no retarding field was used in the later experiments. Nevertheless the current dropped at the critical points, just the same as when the small retarding field was employed. This seems surprising, but the slow electrons produced may not diffuse to the plate as readily as do 20 volt electrons, or the drop in current may be the result of a small contact retarding voltage between the plate and grid $G_2$, or positive ions of an impurity produced by impacts of the second kind with excited helium atoms may diffuse to the plate. If the latter explanation should be the correct one, then both the current drop observed in the Franck method and the rise in current measured in the Lenard method are due to positive ions reaching the plate. At any rate, since the methods employed give results for helium in agreement with spectroscopy, they are thought to be practical and can be used to determine the critical potential of other substances for which no spectroscopic data exist.

*Helium as a calibrating gas.* These experiments show that the use of helium as a calibrating gas for resonance and ionization potential measurements as used by Hertz[7] is justified, since the corrected experimental value is 19.85 volts for the first resonance as compared with the spectroscopic value of 19.77 volts. This difference may be due to experimental error. Or it might be thought in accordance with the suggestion of Dymond that the maximum efficiency of resonance occurs at a voltage higher than the true critical potential, thus causing the peaks to come at higher voltage in these experiments than the spectroscopic values. However, Sponer[4] pointed out that it is possible, as in the present case, to apply *one* correction to a volt-scale and bring all critical points in agreement with spectroscopy. This means that either all critical points have a maximum of efficiency right near the critical point or that such a maximum is displaced the *same* volt amount for each critical potential. The first alternative seems the easier one to believe.

Furthermore, Hertz[7] has used helium as a calibrating gas to determine the resonance points for the other rare gases. In the case of neon he found the critical points in terms of helium using its spectroscopic values. Later he[8] actually obtained the neon resonance lines by Lyman's spectroscopic methods. The two methods checked. If the helium peaks were

[7] Hertz, Zeits. f. Physik **31**, 463 (1925).
[8] Hertz, Zeits. f. Physik **32**, 933 (1925).

displaced a certain amount due to the resonance-maximum appearing a few tenths of a volt higher than the true critical potentials, the calibration would have been incorrect and the two methods could not have checked. It seems, therefore, reasonable to assume that the efficiency of resonance is appreciable a few hundredths volt above the true critical potential.

*The difference between the first and second critical point.* Dymond[5] finds by a new differential method that the difference between the first and second critical potential in helium is 1.0 volt and not 0.8 volt, as is obtained from spectroscopy in the visible and also by Franck by his methods of electron impact. This new result is ascribed by Dymond to the fact that the maxima of resonance efficiency for the two types of impact occur at such electron energies so as to cause this larger difference. However, from Table V it is seen that the present experiments agree with the correct spectroscopic value of 0.78 volt.

*The difference between single and double impacts.* If the possible double transitions are arranged in order of energy required to produce them as in Table IV, it is seen that the difference between the most prominent peak at 40 volts and the first peak near 20 volts is equal to the transition $1S \rightarrow 2P$. The types $AA$ and $AB$ have only been obtained at the highest pressure used (Fig. 2 $E$, 16), whereas the value 40.96 volts (corrected) has been obtained consistently. Because this experimental value checks numerically with the type $CA$, it is ascribed to this transition. In order to understand why this transition should be more probable than the others, the relations between probability of impact and energy of impinging electron must be known. Dymond[5] has pointed out that the probability of double impact is the product of the probabilities of the single impacts at the voltage considered. At 40 volts the probability of type $C$ impact may well be larger than of type $A$ or $B$, and since at 20 volts the probability of type $A$ is larger than of type $B$ the present result would follow. Conversely it may be deduced from these experiments that near 40 volts the probability of impact for type $C$ is greater than for $A$ or $B$.

In conclusion I wish to thank Professor R. C. Tolman for his interest in this research.

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December 23, 1925.

# CHARACTERISTICS OF SHIELDED-GRID PLIOTRONS

By Albert W. Hull and N. H. Williams

Abstract

The internal "feed-back" from plate to grid of a pliotron is prevented by electrostatically shielding the two from each other. Specially designed tubes, thus shielded, are described. The *capacity* between plate and control grid was found to be only .006$\mu\mu$F in the best models. These tubes may, therefore, be expected to behave as strictly unidirectional devices at all frequencies, which was found to be true. *Plate-resistance* characteristics are given, and it is shown that the internal plate conductance is negligible compared with the conductance of practical circuits. The plate current thus depends on grid voltage only, i.e., in place of the seven parameters of the three element tube there is but one, mutual conductance.

## 1. Introduction

THE tubes described in this paper were specially constructed double-grid pliotrons, whose parts were so arranged that the control grid was thoroughly shielded from the electrostatic influence of the plate. They differ from the "screen-grid" tubes described by Schottky[1] only in arrangement of parts, which is such as to minimize electrostatic capacity between plate and control-grid.[2] This difference is unimportant for ordinary low frequency applications, but is essential to high frequency operation, and makes practical many applications that have hitherto been considered impossible. Some of these applications will be described in a separate paper. The purpose of the present paper is to describe the physical and electrical characteristics of the tubes, which are sufficiently unique, it is believed, to be of interest.

## 2. Construction

The ideal aimed at is to prevent, so far as possible, any lines of force passing from the plate or its connecting wires to the control-grid or its connecting wires. This is accomplished mainly by electrostatic shielding, which is facilitated by appropriate methods of supporting the electrodes. Figs. 1 and 3 show experimental structures that have been found to give satisfactory characteristics.

The shielding consists of two parts: (1) The grid itself is shielded from the plate by an interposed grid, maintained at a constant positive

[1] W. Schottky, Archiv. f. Electrotechnik **8**, 299–328 (1919).

[2] Schottky points out (*l. c.*, p. 320) that the screen-grid tubes which he described are not suitable for high frequency amplification.

potential, as in the tubes already familiar.[1,3] Sufficient screening for most circuit conditions may be obtained with wire grids, but measurements show that the wires must be so close together as to intercept a considerable fraction of the electrons, thus reducing the mutual conductance. By making the screen grid of thin, wide slats, placed edge on toward the filament,[4] the shielding may be made as nearly perfect as desired, without offering appreciable obstruction to the passage of electrons. (2) The grid-supporting and leading-in wires must be effectively screened from the plate and its associated wires. This may be accomplished by surrounding them by a grounded conductor as shown in Fig. 1, a structure which was

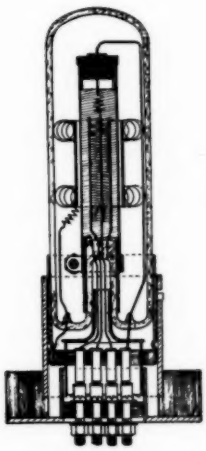

Fig. 1

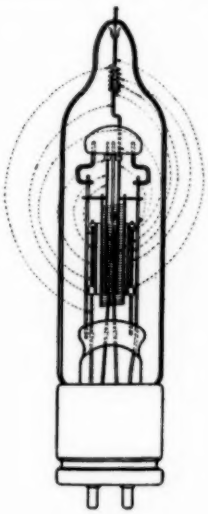

Fig. 2

Fig. 3

Fig. 1. Design of "single-end" shielded grid pliotron.

Fig. 2. Design of shielded-grid pliotron with control-grid terminal at top. The shielding is incomplete, as indicated by the dotted lines, which represent lines of force from the plate to the supporting wires of the control grid.

Fig. 3. Design of completely shielded pliotron, with control grid terminal at top. The shielding grid is supported at the top by a metal disc, which is connected to a metal coating on the inside wall of the tube.

suggested and successfully tried by Mr. D. T. Simonds of this laboratory. This method of shielding has the disadvantage of large grid-to-ground capacity. A better method from this standpoint is to support the grid from the top of the tube, taking care that its wires shall nowhere be exposed to those of the plate (Fig. 2). There will still remain a few lines of force, represented in Fig. 2 by dotted lines, from the outside of the plate to the grid leads. Most of these may be intercepted by a grounded

[3] A. W. Hull, Proc. Inst. Rad. Eng. **6**, 5–35 (1918).

[4] Cf Schottky, *l. c.*, p. 318.

metal cylinder fitting closely over the tube. It was found that the few lines which still pass from plate to grid inside the metal cylinder are sufficient to lead to oscillation, unless the screen grid is extended so as to cover the control-grid supports for a considerable distance and the amount of metal in these supports is reduced to a minimum. The simplest and most complete solution found was a metal disc attached to the top of the screen grid (Fig. 3) in conjunction with a metal coating on either the outside or inside wall of the tube. When an inner coating is used it may be connected internally to the screen-grid, as indicated in Fig. 3, or capacity-coupled through the glass wall to an external grounded conductor.

### 3. Static Electrical Characteristics

These are identical with the characteristics of the double-grid tubes previously described.[1,3] They differ from ordinary three element tubes only in plate resistance, which is infinite or negative at plate voltages

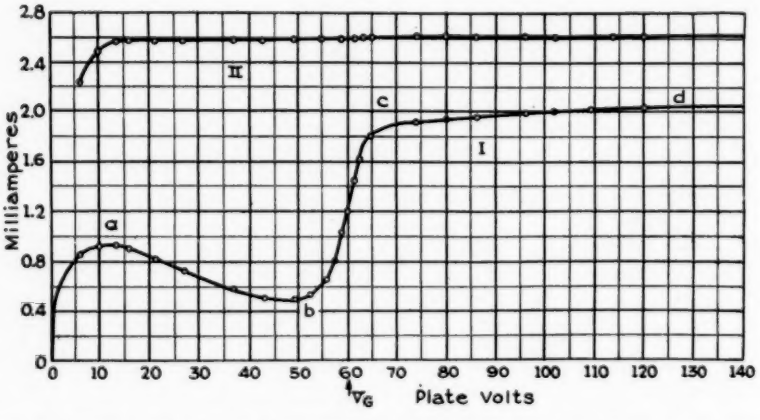


Fig. 4. Plate conductance characteristic of shielded grid pliotron with average secondary emission. The lower curve shows the current to the plate, the upper the total current to plate and screen grid. The potential of the screen grid is indicated by the arrow.

lower than that of the screen grid, positive and very high for voltages above the screen grid. Fig. 4 shows a typical plate characteristic, with screen grid at 60 volts. Curve I in the figure represents the current to the plate, curve II the total current to plate and screen grid. The low value and negative slope of the portion *ab* of curve I is due mainly to the loss of secondary electrons from plate to screen grid, whose number increases with plate voltage. The lowness of the portion *cd*, on the other hand, is due only to electrons intercepted by the screen grid. The finite positive slope of *cd*, as compared with the essentially zero slope of the correspond-

ing portion of the total current curve II, is caused by secondary electrons from the screen grid, only a small fraction of which are able to reach the plate at low plate voltage (70-80 volts in Fig. 4) because of the weak field inside the screen grid, and this fraction increases with plate voltage.

The slopes of both the negative and positive portions of the plate characteristic depend, as is evident from the above description, on the secondary or impact electron emissivity of plate and grid, and the operation of the tubes is greatly influenced thereby. Figs. 5 and 6 show the two extremes of secondary emissivity. The screen grid potential is in both cases 90 volts. Fig. 5 represents a tube whose plate and grid were coated with a thin film of poorly-conducting material, giving high

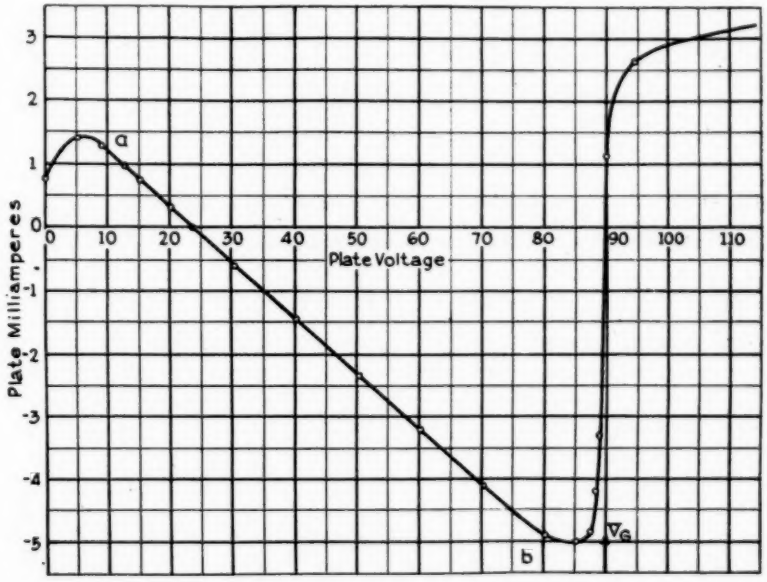


Fig. 5. Plate conductance characteristic of a shielded grid pliotron whose plate has high secondary emissivity.

secondary emissivity. The portion of the characteristic below 90 volts has a negative resistance of −11400 ohms, the portion above 90 volts a positive resistance of approximately 100,000 ohms. Fig. 6 is the characteristic of a tube with a nickel-plated plate, the plating current being adjusted to give a fine black deposit. The secondary emissivity is less than 10 percent over the whole voltage range, and consequently the plate resistance both below and above the screen grid potential is positive and very high, viz., $10^6$ ohms and $7\times10^5$ ohms respectively. It is this characteristic to which attention is especially directed in this paper, since the high plate resistance, combined with low plate-grid capacity, makes this tube a very simple circuit element (see section 5).

The mutual conductance characteristic corresponding to Fig. 4, with plate at 90 volts, is shown in Fig. 7. It is about 30 percent lower than that of a three element tube having the same filament and control grid, and

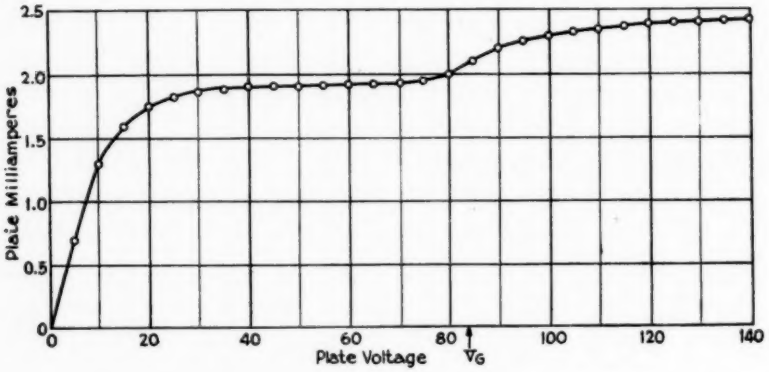


Fig. 6. Plate conductance characteristic of a shielded grid pliotron whose plate has very small secondary emissivity.

a plate at 60 volts in place of the screen grid. This 30 percent loss is due to the electrons intercepted by the screen grid. In other respects this

Fig. 7. Mutual conductance characteristic of tube whose plate conductance is shown in Fig. 4, with shielding grid at 60 and plate at 90 volts.

characteristic is exactly similar to the mutual conductance characteristics of three-element tubes. The same is true of the mutual conductance of the other tubes, such as those represented in Figs. 5 and 6, whatever the

plate voltage. The actual value of mutual conductance is strictly proportional, at any given plate voltage, to the ordinate in the plate resistance curve, Figs. 4, 5, and 6.

### 4. Internal Capacity

The effective capacity between plate and control grid was measured by feeding a signal to the amplifier through the tube, with cold filament. The signal was impressed upon the plate, and the grid was connected to the input of the amplifier, as shown in Fig. 8. A small variable condenser *c* could be substituted for the tube and adjusted to give the same reading of the detector as was observed when the tube furnished the coupling. Care was required to avoid errors due to wiring capacity. This excluded the use of a parallel plate condenser. The condenser which was found satisfactory consisted of a 5 mil (.125 mm) wire in the axis of a 100 mil

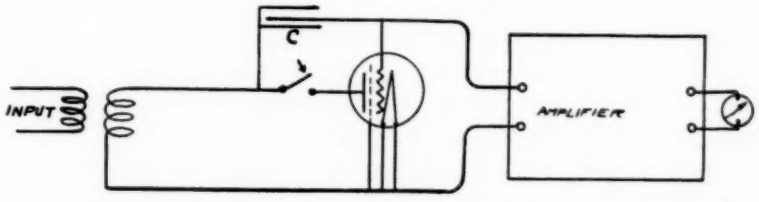


Fig. 8. Circuit used for measuring effective capacity between plate and control grid.

(2.5 mm) cylinder, the wire being movable along the axis. End corrections were avoided by leaving the condenser connected to the amplifier and observing the *increase* in length required to produce the same effect as was produced by connecting the plate of the tube. For a typical wire-grid tube this increase was 2.5 mm, corresponding to a capacity of .025 cm, or .0278$\mu\mu$F. The screen grid, in this case, was a helix consisting of 40 turns per inch of 5 mil (.0125 cm) wire. A tube with a slat type of screen-grid, consisting of thin discs of 3 mm width of section placed 3 mm apart, gave a capacity equal to 0.6 mm length of the cylindrical condenser, or .006$\mu\mu$F capacity. The capacity between plate and grid in standard receiving tubes is between 3 and 6$\mu\mu$F without socket, and from 12 to 40$\mu\mu$F with sockets and wiring. The effective plate-grid capacity has thus been reduced to less than 1 percent of the smallest value hitherto used, in the case of the helical wire screen grids, and to 1/5 of 1 percent in the case of the slat grids.

### 5. The Shielded Tube as a Circuit Element

The circuit behavior of shielded pliotrons is very simple compared with that of ordinary three element tubes. In three element tubes the plate

current at high frequency is a complicated function of seven tube constants. These constants are: the variation of plate current with plate voltage (plate conductance) and with grid voltage (mutual conductance), the variation of grid current with plate and grid voltages, and the three internal tube capacities between plate and grid, plate and filament, and grid and filament respectively.

In the case of well shielded tubes these seven tube constants are reduced to one. Two of the four principal tube "parameters," namely, the variation of plate and grid currents with plate voltage, are made zero (as closely as desired), by the shielding. A third, the dependence of grid current upon grid voltage, is zero when the tube is used as an amplifier with proper C battery bias, such that the grid current remains zero.[5] The plate-grid capacity is made effectively zero by the shielding, and all the other tube capacities become merely circuit constants. The shielded grid amplifier tube has, therefore, but a single parameter, *mutual conductance*. The voltage amplification is the product of this mutual conductance by the plate circuit impedance. The amplification that can be obtained per stage is limited only by the impedance that can be built up, by resonance or otherwise; and the number of tubes that can be usefully operated in series is limited only by the terminal conditions, viz., the condition that the input voltage must not be smaller than the "shot effect" or other inherent input disturbance, and that the amplified output cannot be greater than the plate-battery voltage employed for the last tube. The shielded tube is also a strictly unidirectional amplifier, to any degree of strictness that may be desired, as is evident from the capacity tests reported in the last section.

These predictions have been tested and verified by amplification experiments, which are reported in a separate paper.

It will be understood that the purpose of this paper is scientific, and bears no relation to commercial development. Tubes of the type described are not being manufactured, nor is their immediate production contemplated, so far as the authors are aware.

We gratefully acknowledge the cooperation of Mr. Harry C. Thompson in the design and construction of these tubes.

RESEARCH LABORATORY,
GENERAL ELECTRIC COMPANY.
December 30, 1925.

[5] The charging current flowing into the grid because of the grid-to-ground capacity is a circuit and not a tube parameter, since this capacity is simply an additive part of the circuit capacity. This is not true in unshielded tubes, where the effective grid capacity depends upon plate voltage.

# MEASUREMENTS OF HIGH FREQUENCY AMPLIFICATION WITH SHIELDED-GRID PLIOTRONS

By Albert W. Hull

Abstract

Tests of cascade high frequency amplification are described, using specially constructed tubes in which the control grid is electrostatically shielded from the plate. These tubes are free from internal feed-back, so that any number may be operated in series. The *voltage amplification* per stage depends only on the circuit impedance, and was found to be 200 at 50 kilocycles, 40 at 1000 k.c., and 7 at 10,000 k.c. *Total amplifications* as high as 2,000,000 in voltage were obtained and measured at 1000 k.c. It is shown that this is the largest amplification that can be usefully employed at any frequency. At 10,000 k.c. a total amplification of 10,000 was obtained. Examples are given of other applications, such as the amplification of small photo-electric currents, where shielding is advantageous.

## 1. Introduction

IT IS commonly believed that any electrical impulse, however small, can be amplified by modern thermionic valves to any desired degree. This belief is completely justified. The only limitation that need be imposed is that the duration of the impulse shall not be shorter than the time of transit of an electron across the tube.

The practical use of such amplification is limited by two considerations: The difficulty of constructing and operating the amplifier may be prohibitive; and the signal may not be worth amplifying, because of the presence of other stronger impulses from which it cannot be separated.

It is the purpose of this paper to show how the first of these limitations, the difficulty of building and operating the amplifier, can be avoided by the use of properly constructed tubes.

The difficulty of building amplifiers is due to instability, caused by "feed-back" of electrical energy from the output circuit to the input circuit. If the impulse thus fed back is in opposite phase to the input signal the amplification will be reduced; if in the same phase it will be increased or "regenerated," and sustained oscillations will result if the amount of energy thus fed back is greater than the losses in the input circuit. Feed-back due to *external* causes, such as electrostatic or magnetic coupling between output and input circuits, or resistance or inductance in common battery and ground wires, can easily be avoided. The most common and obstinate cause is *internal* coupling, that is, coupling due to electrostatic capacity between plate and grid in the same tube. The

energy thus fed back through the tube in ordinary receiving tubes is 10,000 times the amount necessary to sustain oscillation in good tuned circuits, at 1000 k.c.

Three practical methods of avoiding oscillation from this internal feed-back have been employed in the past. The first is to use only a single stage of amplification and to so arrange the circuit that the plate voltage is approximately 90° out of phase with the input (grid) voltage. This is the ordinary regenerative method, and is capable of producing from 10 to 100 fold voltage amplification by careful adjustments.

The second method is to use poorly tuned or high resistance circuits, with a small gain per stage. In this way, by keeping the energy fed back per stage smaller than the circuit losses, total amplifications as large as desired may be obtained. This is a practical method, but requires a large number of tubes, since the safe gain per stage is not more than 3 at 1000 k.c., under ordinary circuit conditions, and decreases rapidly with increasing frequency.

The third method is to neutralize the unavoidable feed-back by an intentional feed-back of opposite phase. The two feed-backs must be balanced within about 1 percent at 1000 k.c. with ordinary tubes and good tuned circuits. If such a balance can be found and maintained for each stage there is no limit to the amplification attainable. The difficulty of obtaining exactly opposite phase relations is very great, however, because of the presence of distributed capacity in the tuning coils, and generally limits the total practical voltage amplification obtainable in this way to the order of 100 fold at 1000 k.c., or 1000 fold at 50 k.c.

The method described in this paper is to avoid feed-back entirely by using tubes so constructed that there is no capacity coupling between plate and control grid. It was found that amplification as large as desired could be obtained at any frequency without instability, with a very small number of tubes.

## 2. Description of Tubes and Circuit

The shielded-grid tubes and their characteristics have been described in a previous paper.[1] For convenience of reference Fig. 3 of that paper, which represents the type of tube used for these measurements, is reproduced here (Fig. 1). As seen from this figure, the shielding of the control-grid from the plate is accomplished internally by means of an extra grid, maintained during operation at a potential which is constant with respect to ground; and, externally by appropriate arrangement of supporting and leading-in wires and the use of grounded conductors.

[1] A. W. Hull and N. H. Williams, Phys. Rev. **27**, 432 (1926).

In this way it was found possible to reduce the effective capacity between plate and control grid to less than 1 percent of its value in three-element tubes. This capacity is a measure of the plate conductance, which is therefore reduced in the same ratio and becomes negligible in comparison with ordinary load conductances. The behavior of the tubes is thus greatly simplified as compared with three-element tubes. They become strictly unidirectional valves, whose sole parameter, the change of plate current with grid voltage, is independent of all plate-circuit variables.[2]

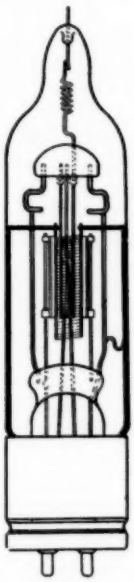

Fig. 1. Drawing of special tube used for high-frequency amplification measurements, showing method of shielding the control grid from the plate.

The circuit used for the cascade amplification tests described below is shown in Fig. 2. It is the ordinary capacity-coupled arrangement, each plate being connected to the 110 volt line through a multiple-tuned circuit $LC$, and to the next grid through a small condenser $c$. Resistances $r$, approximately 1 megohm, maintain the average grid potential at the proper value. The tubes and circuits are mounted in individual boxes covered with tinned sheet iron. These boxes are piled on top of each other, and the top of each tube projects into the box above, making the shielding complete, provided the metal boxes can be maintained at constant potential. Common batteries were used, except as noted below, and the feed to each stage was filtered by a 10 millihenry choke in conjunction

[2] It is assumed that the plate voltage is kept within the operating range.

with one or more 2 m.f. condensers. It was considered essential that the grounded terminals of all the filter condensers in a single box be soldered to the box at the same point, in order to minimize circulating currents in the box walls. The screen-grids were maintained at 60 volts, unless otherwise stated, and the plates at 110 volts. For convenience in tuning and in measuring amplification, each stage was supplied with a jack, and the method of coupling allowed any tube to be used as detector. In

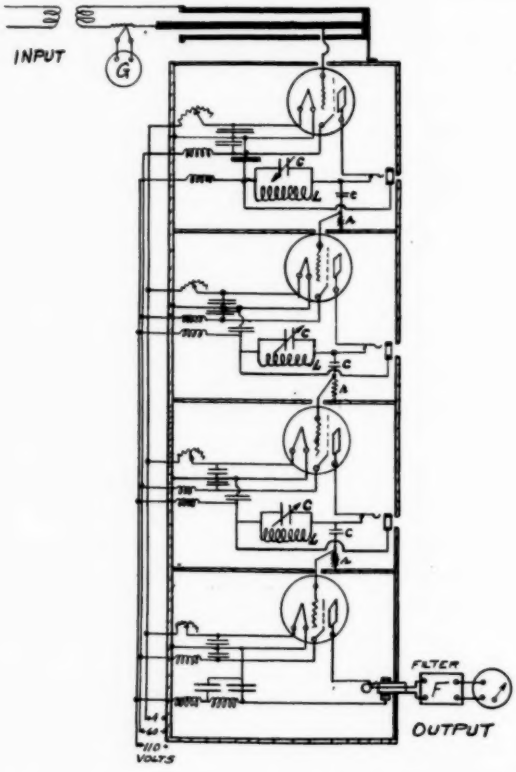


Fig. 2. Diagram of cascade amplifier used for tests at 1000 and 10,000 kilocycles.

this way the performance of each tube could be checked, and successive stages tuned in, one by one.

The amplification referred to in the following tests is the voltage amplification due to the tubes only, defined as the ratio of r.m.s. alternating voltage of the plate to that of the grid, or of the last plate to that of the first grid in case of more than one tube. Output voltages were obtained in the usual manner by calibration of the detector. The small input voltages required were obtained from the inductive drop in a short section of straight copper rod surrounded by a concentric return

conductor, as shown in Fig. 2. For frequencies up to 1000 k.c. this method is reliable and free from difficulties; at 10,000 k.c. care is required in connecting the input system to the amplifier, to avoid capacity-charging currents in the grounding and shielding conductors.

### 3. Amplification Measurements at Low Radio Frequency (50 k.c.)

Preliminary measurements were made to determine the best type of plate circuit impedance. It was found that no advantage was derived from a step-up transformer of any type. This might have been anticipated, since the high p'ate resistance of the tube enables it to work efficiently into the highest obtainable impedance. Single tuned circuits were, therefore, used, with condenser feed to the next tube, as shown in Fig. 2. Measurements with coils of different numbers of turns showed, as expected, that the maximum amplification was obtained with the highest possible value of inductance, since this gives the highest impedance.

At low frequencies it is possible to build resonant circuits of very high impedance. Since the voltage amplification is the product of this impedance by the mutual conductance of the tube, high amplifications may be expected. A few measurements at 50 kilocycles showed that a voltage amplification of 200 fold per tube could easily be obtained. The stability and limiting value of multistage amplification was not investigated, but it may safely be assumed from the results at 1000 k.c. that there is no instability at these low frequencies. The measurements of Johnson[3] have shown that there are no tube disturbances at these frequencies except "shot effect," hence the maximum useful amplification is limited only by this ever-present restriction. This limitation is discussed in section 7 below.

### 4. Medium Frequencies (1000 k.c.)

The circuit shown in Fig. 2 was used. The tubes were first tested singly, and gave voltage amplifications between 40 and 45 at 750 k.c. When operated in series, two tubes gave 1700, and three tubes 75,000, which is equal to the square and cube respectively of the amplification of a single tube, within the limit of accuracy of the measurements. There was no evidence of regeneration. These measurements were made with low impedance input. The set was later used with high impedance tuned input and found to be stable.

A second series of measurements, at 1000 kilocycles, was made with a set containing 6 shielded grid tubes, similar to that shown in Fig. 2. The tubes amplified approximately 40 fold each. Under measuring

[3] J. B. Johnson, Phys. Rev. **26**, 71 (July, 1925).

conditions, i.e., with low impedance input circuit, the first four tubes operated stably in series (the fifth being used as detector) and gave a total voltage amplification of 2 million. This corresponds to 38 fold per stage. All circuits were tuned to exact resonance, and the amplified voltage was found to vary steadily and symmetrically with the tuning of each one of them. The degree of amplification was independent of input voltage from 1 microvolt, the smallest signal that could be distinguished from the "shot effect," to 10 microvolts, which filled the operating range of the last amplifier tube.[4]

The selectivity curve of the four tubes in series (input untuned) is shown in Fig. 3. It coincides very closely with the curve calculated from the circuit constants.[5]

In the first tests with the 4 stage amplification described above it was observed that oscillations resulted when a tuned circuit was substituted for the low-impedance input shown in Fig. 2. The cause of this feedback was found to be external to the amplifier, namely in the common battery supply. The oscillations and all signs of instability disappeared when a separate battery supply was used for the first two stages. The

[4] It was obviously necessary, in order to avoid saturating the detector, either to use a detecting tube with very low amplification constant, which need not, however, have a shielding grid; or to operate the detector as a tube volt-meter, with variable negative grid bias, adjusted at each reading to give a definite small plate current.

[5] Since mutual inductance is independent of frequency, the theoretical selectivity curve for a single stage is simply the curve of plate impedance, and for $n$ stages the $n$th power of this impedance. The impedance of a single tuned circuit, viz.

$$z_\omega = \frac{L^2\omega^2 + R^2}{\sqrt{(C\omega[L^2\omega^2+R^2]-L\omega)^2+R^2}}$$

reduces, for practical radio circuits, in which $R^2$ is always negligible compared with $L^2\omega^2$, and for the small values of $\omega-\omega_0$ which are of interest, to

$$Z_\omega = \frac{L^2\omega^2}{\sqrt{4L^2(\omega-\omega_0)^2+R^2}} \qquad (\omega_0 = \text{resonance frequency})$$

The relative impedance is

$$\frac{Z_\omega}{Z_0} = \frac{R}{\sqrt{4L^2(\omega-\omega_0)^2+R^2}} = \frac{R}{\sqrt{157.9L^2(f-f_0)^2+R^2}}$$

where $Z_0$ is the impedance at the resonant frequency $\omega_0/2\pi = f_0$.

If $y$ is the ordinate of the selective curve, that is, the relative voltage amplification, then for one stage of amplification

$$y = \frac{Z_\omega}{Z_0} = \frac{R}{\sqrt{158L^2(f-f_0)^2+R^2}}$$

and for $n$ stages,

$$y_n = \frac{R^n}{[158L^2(f-f_0)^2+R^2]^{n/2}}$$

same result could undoubtedly have been accomplished with a single common battery supply by better filtering.

Similar results were obtained with 5 stages of amplification. In this case the amplification per stage was reduced, by using coils with smaller inductance, so that the total voltage amplification was still two million, which was the largest that could easily be measured. There was no evidence of instability, with all circuits, including the input, tuned in any desired manner.

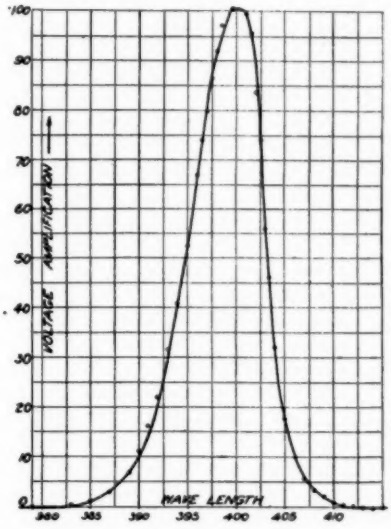


Fig. 3. Selectivity curve of 4 stage amplifier with input untuned, at 1000 k.c. The voltage amplification is $2\times10^6$.

The observed amplification of 2 million fold in voltage is very near the maximum value that can be usefully employed, because of the presence of shot effect. (This limitation is more fully discussed in section 7.) These tests show, therefore, that any degree of amplification which may be desired can be obtained at 1000 kilocycles, provided external circuit feed-back is avoided.

## 5. High Frequencies

At high frequencies dielectric and resistance losses are so large that it is not easy to obtain high impedance by resonance, hence the maximum amplification per stage is smaller than at lower frequencies.

Two series of measurements at 10,000 kilocycles (30 meters) were carried out with the 6 tube amplifier described above. In the first series the same condensers (31 plates, 1 mm apart) and same wiring

connections, including jacks, were used as in the medium wave-length tests, the coils only being changed. The losses were high, and the impedance at resonance only 8000 ohms (calculated from the ratio of amplification to mutual conductance). This gave a voltage amplification of only 3 per stage. There was no instability, and five stages in series could be operated very simply, with a total amplification of 250.

In the second series of tests, special low loss coils were used, each consisting of 10 turns of 1/4 inch copper tubing, wound in a self-supporting helix of 3 inches diameter and 3 inches total length. The same 31 plate condensers were used (at nearly minimum capacity) but the jacks were by-passed and other insulation losses minimized. The circuit impedance was thus increased to over 16,000 ohms, and the voltage amplification to nearly 7 per stage. It was found that 5 stages in series were still stable, giving a total voltage amplification between 10,000 and 15,000. A series of measurements under these conditions is given in Table I. The variations are due to lack of care in turning, as high accuracy was not sought. The tuning was sharp, but uniform and symmetrical.

TABLE I

*5 stage amplification at 10,000 kilocycles*
(Circuit like Fig. 2, except jacks. Plate voltage 110, screen voltage 65)

| Length of input Inductor 1cm = .00358μh | Current through Inductor | Input volts | Detector reading | Output volts | Voltage amplification |
|---|---|---|---|---|---|
| (cm) | (milliamps) | | (milliamps) | | |
| 0 | | | 4.42 | 0 | |
| 3 | .37 | $.25\times10^{-3}$ | 4.02 | 2.5 | 10000 |
| 3 | .47 | .32 | 3.85 | 3.8 | 12000 |
| 3 | .82 | .55 | 3.45 | 7.3 | 13300 |
| 3 | 1.10 | .74 | 3.30 | 9.0 | 12200 |
| 3 | 1.66 | 1.12 | 3.05 | 12.0 | 10700 |
| 9 | .32 | .64 | 2.85 | 16 | 25000 |
| 9 | .37 | .74 | 2.95 | 13.6 | 18400 |
| 9 | .42 | .84 | 2.90 | 14.8 | 17600 |
| 9 | .80 | 1.6 | 2.80 | 18 | 11200 |
| 9 | .92 | 1.84 | 2.65 | 23 | 12500 |
| 12 | .32 | .86 | 2.95 | 13.6 | 15800 |
| 12 | .37 | 1.00 | 2.90 | 14.8 | 15800 |
| 12 | .80 | 2.15 | 2.85 | 18 | 7500 |

Tuning the input circuit under these conditions caused oscillation. Since these tests were made with the same common battery supply which led to oscillation at 1000 k.c., it is believed that the cause of oscillation in this case was the same, namely insufficient filtering of the power supplied from the common batteries to the individual tubes. The degree of instability was about the same as at 1000 k.c. and could pre-

sumably be cured in the same way, by separate battery supply. This has not been tried.

There is every reason to believe that still shorter wave-lengths could be amplified directly in the same manner. The simplest procedure with short waves, however, is to allow them to beat with a local oscillator to give a difference frequency of 10,000 k.c., which can then be amplified as just described. In this way it should be possible to amplify wave-lengths as short as 2 or 3 meters, with no greater delicacy of adjustment than is required to amplify wave-lengths of 400 to 600 meters with the customary intermediate frequency of 50 kilocycles.

## 6. Discussion of Stability

The conditions under which oscillation can occur from internal capacity feed-back are easily seen from the simplified circuit shown in Fig. 4, which is the a-c. equivalent of Fig. 2. Each plate is capacity coupled to the control grid of the same tube by an effective capacity $C_0$.[6] Since these capacities are all in series, the $n$th plate is coupled to the first grid by a capacity $C_0/n$. This capacity decreases only arithmetically with successive stages, while the amplification increases geometrically. It is evident, therefore, that a stage must eventually be reached, if there were no attenuation, where the energy fed back from the last tube to the first would be greater than the input circuit losses.

The energy thus fed back is attenuated at each stage by the losses of the tuned plate circuit, which includes the capacities to ground of the plate and grid, the grid leak resistance, and the internal plate-resistance of the tube. This is equivalent to a well-known type of filter circuit, whose attenuation may be calculated by the methods described by Campbell and others,[7] provided the terminal impedances are properly adjusted. In the present case a simpler procedure is sufficient. The problem is this: Given a series of identical tubes and circuits, each tuned to approximate resonance, what is the maximum amplification per stage at which self-sustained oscillations are impossible. The only phase relations that need be considered are those favorable to maximum regeneration, since the phases of tuned circuits change so rapidly with

[6] This is called "effective" capacity because it is not a capacity which can be measured in an ordinary capacity bridge. It may be defined as the number of lines of force from the plate that end on the grid per unit potential difference; or as the equivalent ordinary capacity that would have the same alternating-current admittance.

[7] G. H. Campbell, Bell. Sys. Tech. J. **1** (Nov. 1922);
O. J. Zobel, Bell Sys. Tech. J. **2**, 1–47 (Jan. 1923); **3** (Oct., 1924);
L. Cohen, J. Frank Inst. **195**, 641–54 (1923).

tuning that this condition can hardly be avoided, when dealing with more than one stage. This condition is satisfied by exact resonance, which gives the maximum gain per stage, and brings the voltage of each plate in phase with that of the grid of the preceding tube. The plate impedances are then pure resistances, $R$. Denoting the voltages at

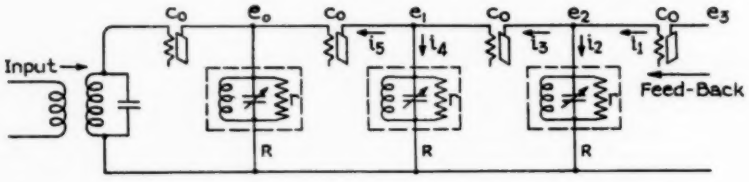


Fig. 4. Simplified diagram of circuit of Fig. 2, showing feed back.

successive stages by $e_1$, $e_2$, $e_3$ (Fig. 4) the currents in the various branches are given by the following equations:

$$i_1=(e_3-e_2)C_0\omega$$
$$i_2=e_2/R$$
$$i_3=(e_2-e_1)C_0\omega$$
$$i_4=e_1/R$$
$$i_5=e_1/\sqrt{R^2+(1/C_0\omega)^2} \qquad \text{(approx.)}$$
$$i_1-i_2-i_3=0$$
$$i_3-i_4-i_5=0$$

For all cases of interest $R^2$ may be neglected in comparison with $(1/C\omega)^2$. Elimination of $e_2$ and the five currents between these equations then gives for the relative voltage of the first grid due to feed-back from the third:

$$e_3/e_1=(1/RC_0\omega+2)^2-1 \qquad (1)$$

The condition for stability is that this attenuation ratio shall be greater than the two stage amplification. If $\mu$ is the amplification per stage, this condition is

$$e_3/e_1=(1/RC_0\omega+2)^2-1>\mu^2$$

or

$$\mu<1/RC_0\omega+2 \qquad (2)$$

with sufficient approximation.

The resistance $R$ may be expressed in terms of amplification and the mutual conductance $g_m$ and plate resistance $r_p$ of the tube:

$$\mu=\frac{g_m}{1/R+1/r_p} \tag{3}$$

Substituting this value of $R$ in Eq. 2, one obtains for the maximum stable amplification,

$$\mu<\sqrt{g_m/C_0\omega+(1/2r_pC_0\omega-1)^2}-1/2r_pC_0\omega+1 \tag{4}$$

For screen grid tubes $1/r_p$ is negligible, and Eq. 4 becomes

$$\mu<\sqrt{g_m/C_0\omega+1}+1 \tag{5}$$

In the above tests $g_m=400\times10^{-6}$, $C_0=.028\times10^{-12}$. At 1000 k.c. ($\omega=6.3\times10^6$) Eq. 5 gives $A<49$. The actual amplification per stage was between 40 and 45. Hence the condition was a stable one, but the margin of stability was small. At 10,000 k.c. ($\omega=63\times10^6$) Eq. 5 gives $A<19$. The maximum amplification actually used was 7. To attain an amplification of 19 would require a load impedance of 45,000 ohms, which is probably higher than can be obtained in practical circuits at this frequency. If this is so, the amplifier would be stable at this frequency under all load conditions, provided external sources of feed-back are avoided.

These calculations show that the plate-grid capacity of these tubes, although it has been reduced by shielding to less than 1/100 that of the best three electrode tubes, is barely small enough for stable high frequency amplification. A value 1/4 as great, such as was obtained by the use of a slat type of screen grid,[1] and such as can be obtained with wire grids with a slight sacrifice in mutual conductance, would be ample under all circuit conditions at any frequency.

According to Eq. 4, which is applicable to any kind of tube, the maximum stable amplification with 3 element tubes having a mutual conductance of 400$\mu$mhos and plate resistance 30,000 ohms, and with the minimum plate-grid capacity of 2.5$\mu\mu$F, should be 5 at 1000 k.c. Space charge grid tubes, with 4 times the mutual conductance of the single grid tube and the same plate-grid capacity and plate resistance, should be capable of nearly ten fold per stage without instability. The attainment of these values requires special wiring. With ordinary sockets and wiring the total plate-grid capacity is seldom smaller than 12$\mu\mu$F, which reduces the possible stable amplification for three-element tubes at 1000 k.c. to 2.4, and for the space-charge grid tube to less than 5.

With 3-grid tubes,[8] utilizing the advantages of both space-charge and screen grids, the maximum stable voltage amplification per stage at

[8] Cf W. Schottky, Archiv. f. Electrotechnik **8**, 321 (1919).

1000 k.c. should be over 100, according to Eq. 5, for a tube with plate-grid capacity of .025, and over 200 for .006$\mu\mu$F capacity. At least 3/4 of this higher value, viz., 150, should be capable of realization with practical circuits such as were used in the above tests.

### 7. Limit of Useful Amplification, Due to Shot Effect

It was observed in the tests at 1000 k.c. that the detector tube showed a small signal even when none was impressed upon the input circuit. This spontaneous generation of oscillations, which is known as "shot effect,"[9] has already been observed and studied under circuit conditions identical with those here used.[10] It is due to shock excitation of the plate circuits—principally that of the first tube, which receives the greatest amplification—by the individual electrons that constitute the plate current. The total r.m.s. voltage thus produced, which is the vector sum of these elementary shock excited voltage oscillations, depends on their total number, that is, on the plate current, on the charge of the electron, on the capacity and power factor of the circuit, and on the degree of space charge. For ordinary radio-frequency tuned circuits, and emissions such as are used in receiving tubes, this r.m.s. voltage lies between 10 and 100 microvolts.

The existence of this shot effect sets a limit to total useful amplification. Amplification ceases to be useful when the shot voltage originating in the first tube fills the detector. For example, if the first tube, in order to amplify 40 fold, must use a plate current so large and circuit damping so small that its shot voltage is 20 microvolts, then with 2 million fold total amplification the amplified shot voltage will be $(2\times10^6/40)\times20\times10^{-6}=1$ r.m.s. volt.[11] This is nearly the maximum amplitude that can be handled by ordinary detecting tubes. It can easily be shown that, for a given degree of selectivity, the ratio of amplified signal to shot noise is independent of circuit conditions, and depends, for a given signal, only on the mutual conductance of the first amplifier tube. The above example was calculated for a mutual conductance of 400$\mu$ mhos, which is an

[9] W. Schottky, Ann. d. Physik, **57**, 541–67 (1918); **68**, 157–76 (1922).

[10] A. W. Hull and N. H. Williams, Phys. Rev. **25**, 147–73 (1925).

[11] The actual value observed in the above tests was 1.3 volts. When a signal is added to this the energies add, so that the observed voltage is the square root of the sum of squares of signal and shot voltage. As an example, a sine voltage of 2 microvolts impressed upon the grid of the first tube gave an amplified voltage reading at the grid of the detector of 4.8 volts. A deflection corresponding to 1.32 volts was observed with no signal, due to the shot effect. The amplified voltage due to the signal of 2 microvolts was, therefore, $\sqrt{(4.8)^2-(1.32)^2}=4.6$ volts. In this case, in order to distinguish the signal from shot effect, it was necessary to use a detector tube with the unusually large range of 4.8 volts.

average value for receiving tubes. Hence the limit of useful amplification with ordinary tubes may be taken as a million fold in voltage. With a special input tube of high mutual conductance, and a low impedance detector, a maximum of 10,000,000 fold may be utilized. These limits depend upon no fault of tube or circuit, but upon the charge of the electron.

The above maximum values refer to tube amplification, beginning with the grid of the first tube. Any increase of signal voltage *before* it is impressed upon the amplifier, by transformation or resonance, is added gain.

At frequencies below 10 k.c. there are other disturbances, originating in the tubes and circuits, which are from 10 to 100 times larger than the shot effect.[3] Hence at these low frequencies, namely in the audible and sub-audible range, the maximum useful amplification is between 10,000 and 100,000.

## 8. Other Applications

The above description of cascade amplification has been given in full because the results, especially at the higher frequencies, can be obtained practically in no other way. There are other problems, which, though soluble with ordinary tubes, are greatly facilitated by the use of shielded tubes. Two examples will be given.

The first example is the amplification of small rapidly varying currents, such as the photo-electric currents produced by light from successive small areas of a picture, that is to be transmitted by wire or radio. The limit to the speed with which such small aperiodic currents can be reproduced is set by the total effective grid-to-ground capacity, which must be charged or discharged in order to change the grid potential. In the ordinary method of amplifying such currents, the photo current is made to flow through high resistance $r$ (Fig. 5) and the resulting potential difference $e$ at the terminals of $r$ is amplified. The maximum value of $r$ for any given frequency is determined by the capacity of the system, as follows: The photo current $i$ flows through the resistance $r$ and effective grid-to-ground capacity $C$ in multiple, and is given by

$$i = i_r + i_c = e/r + C\, de/dt$$

hence $de/dt + e/Cr = i/C$. Solution of which is:

$$e = ir(1 - \epsilon^{-t/Cr}) \qquad (1)$$

$e$ will therefore reach 2/3 of its final value $ir$ in a time given by $t/Cr = 1$. (2)

If $t$ is given, the *maximum* allowable value of $r$ is thus determined, viz.

$$r=t/C \tag{3}$$

The capacity $C$ is made up of three parts in multiple, viz., the anode-cathode capacity of the photo cell, the grid-filament capacity of the

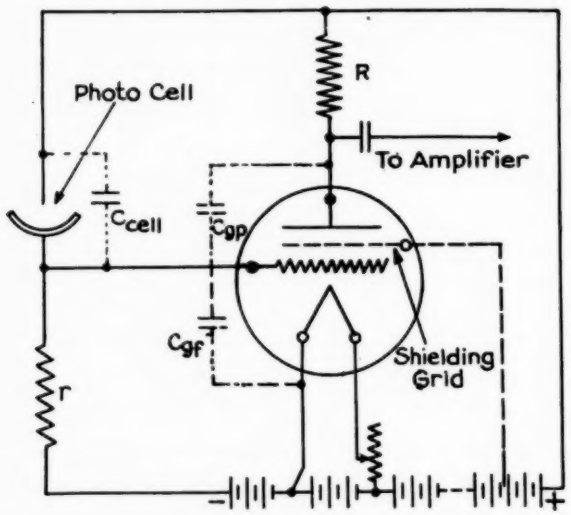


Fig. 5. Circuit for amplifying photoelectric currents, showing advantage of shielding.

pliotron, and the effective grid-plate capacity of the pliotron. The latter is $\mu$ times the actual grid-plate capacity, where $\mu$ is the actual amplification.

Hence $C=C_{cell}+C_{gf}+\mu C_{gp}$

or, taking the three measured capacities as approx. equal

$$C=(\mu+2)C_{Pg} \tag{4}$$

Substituting these values of $r$ and $C$ in Eq. (1) gives for the maximum voltage at the grid

$$e_g=it/(\mu+2)C_{gP} \tag{5}$$

The amplified voltage at the plate is $\mu$ times this, viz.,

$$e_p=\mu e_g=\mu it/(\mu+2)C_{gP} \tag{6}$$

It is seen that nothing is gained by amplification, since the fraction $\mu/(\mu+2)$ increases but little with $\mu$. In other words, the photo-electric current $i$ cannot be amplified appreciably by the first amplifier tube, provided it is of the three-element type. This sets a limit to the smallness of the non-sinusoidal current $i$ which can be amplified at any given

frequency $1/t$, since the voltage at the *plate* of the first amplifier tube must not be smaller than the shot voltage originating in this tube.

If a shielded-grid amplifier is substituted for the three-element tube, as indicated in Fig. 5, the effective grid-ground capacity is simply its measured capacity, and instead of the fraction $\mu/(\mu+2)$ in Eq. (6) we have approximately $\mu/3$. Since values of $\mu$ as high as 100 are easily attainable with shielded tubes at the frequencies under discussion, the amplified voltage $e_p$ will be more than 30 times that obtainable with a three element tube, and the minimum current that can be reproduced will be correspondingly increased.[12]

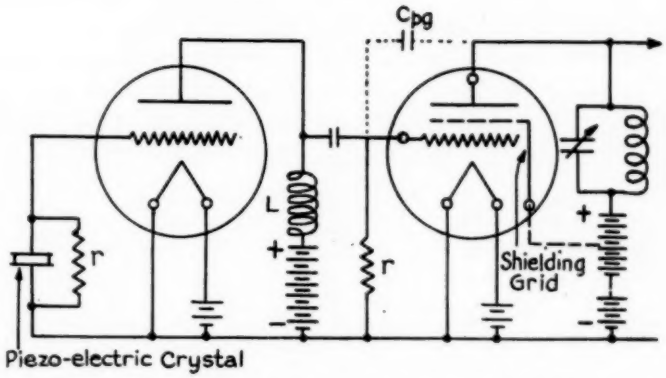


Fig. 6. Crystal-controlled oscillator circuit, showing use of shielded-grid amplifier to prevent change of frequency.

A second example of a useful, but not essential, application of shielded grid tubes is in connection with the utilization of exact standards of frequency for the measurement or control of other frequencies. It is well known[13] that a piezo-electric crystal, such as quartz, may be used to control the frequency of a vacuum tube oscillator, giving extreme constancy. With proper choice of circuit constants and with the crystal maintained at constant temperature, the frequency remains constant to less than 1 part in a million, independent of all ordinary fluctuations of supply voltage, or such changes of circuit constants as are incident to temperature variation or chance mechanical movements.

In order to use such a standard it is necessary to couple it to another system, which will generally react upon the standard and alter its

[12] The gain in minimum reproducible current will not be quite 30 fold, since, if all attempts to amplify with the three element tube are abandoned, the resistance in its plate circuit can be made very low, which will somewhat reduce the shot effect.

[13] W. G. Cady, Phys. Rev. **19**, 1 (1922); Proc. Inst. Radio. Eng. **10**, 83 (1922); **12**, 805 (1924).

frequency. The reaction is smallest when the standard is loosely coupled to the grid of another tube, as shown in Fig. 6. Even in this case, however, tuning the plate circuit of the second tube will in general change the frequency of the standard by 10 to 100 parts in a million, due to the internal capacity reaction between plate and grid of the second tube. This reaction may be balanced by an equal reaction of opposite phase, in the well-known manner, and the effect on the standard circuit thus reduced. But the degree to which this has been accomplished can be ascertained only by comparison with another standard. By using a shielded tube for the second amplifier, as indicated in Fig. 6, all danger of reaction may be avoided, and the frequency made independent of the tuning of the utilization circuit to any desired degree.

It will be understood that the purpose of this paper is scientific, and bears no relation to commercial development. Tubes of the type described are not being manufactured, nor is their immediate production contemplated, so far as the author is aware.

RESEARCH LABORATORY,
GENERAL ELECTRIC CO.,
December 30, 1925.

# THE ELECTRIC MOMENT OF $CO_2$, $NH_3$, AND $SO_2$

By C. T. Zahn

Abstract

**Dielectric constant of $N_2$, $CO_2$, $NH_3$ and $SO_2$.**— By an improved heterodyne null method previously described, measurements were made of the dielectric constant at atmospheric pressure of the carefully purified gases, over a range from 300° to 400° from just above the liquefaction point. These results as well as all others found by the author are represented by the Debye equation $(\epsilon-1)vT=AT+B$, where $\epsilon$ is the dielectric constant, $T$ the absolute temperature, and $v$ the specific volume. The values found for $A$ are .000958 ($CO_2$), .000768 ($NH_3$), and .001433 ($SO_2$); and for $B$ are .003 ($CO_2$), 1.740 ($NH_3$), and 2.167 ($SO_2$). The anomaly observed by Jona and attributed to association was not observed except possibly to a slight degree in $NH_3$. The values of the *electric moment of the molecules* calculated from the constant $B$ differ from Jona's values by 7 percent. It is believed that the explanation of the difference is to be based on the fact that the anomaly is not due to association but to surface effects on the metallic plates of the condenser. The values obtained for the electric moment are, in c.g.s., e.s. units$\times 10^{18}$, .06 ($CO_2$), 1.44 ($NH_3$), and 1.611 ($SO_2$). The small value for $CO_2$ differs from zero by an amount which is less than the experimental error.

IN an article by Jona[1] on the variation of the dielectric constant of gases with temperature marked deviations from the Debye straight line were recorded in the cases of water vapor, methyl alcohol, and sulfur dioxide near the liquefaction point. These deviations were attributed to association with an accompanying increase in the electric moment of the molecule. In a series of measurements[2] on the halogen hydrides no such deviations from the Debye theory were observed even though the measurements were made near the liquefaction points of the gases studied. For this reason it was thought worth while to make a further investigation of this anomalous effect. To this end measurements were made on several of the gases studied by Jona.

The experimental method used has been described by the author in the above mentioned paper[2] on the electric moments of the halogen hydrides. A heterodyne null method is used in which the beats between the oscillations from two separate electron-tube generators operating at frequencies of about 1,000,000 and 1,001,000 cycles per second are adjusted to the frequency of an electrically driven tuning fork, the small change of capacity due to the introduction of the gas into the measuring

[1] M. Jona, Phys. Zeits. **20**, 14 (1919).
[2] C. T. Zahn, Phys. Rev. **24**, 400 (1924).

condenser being compensated by a large change in a large capacity in series with this condenser.

According to Debye's theory[3] the dielectric constant of a gas is given by the equation

$$(\epsilon-1)vT=AT+B$$

when $\epsilon$ is the dielectric constant, $T$ is the absolute temperature, and $v$ is the specific volume referred to the volume occupied by 1 cc of an ideal gas at 0°C and 760 mm Hg. For an ideal gas $v=T/273$ but for vapours this is corrected by the use of van der Waals' equation. $A$ is a constant which represents the electric polarization due to induced distortion in the molecule and $B$ is a constant which represents the orientation polarization and is proportional to the square of the electric moment of the molecule.

$$(\mu^2=1.198\times10^{-36}B)$$

If $(\epsilon-1)vT$ is plotted against $T$ there is obtained a straight line of slope $A$ and intercept $B$ which determines $\mu$. For gases of symmetrical structure, i.e., zero electric moment, $B=0$ and the straight line should pass through the origin of coordinates.

Measurements were first made on $N_2$ as a rough check on the behaviour of the apparatus. Since $N_2$ is not a polar molecule one should expect zero electric moment for the molecule and the Debye straight line should correspondingly pass through the origin of coordinates. This was found to be the case within the limits of experimental error. It should be noted here that for the measurements on nitrogen the temperature control of the apparatus was not so good as for the other measurements which were made later when an oil bath of large heat capacity was used for the high temperatures. The later measurements are undoubtedly more accurate than those on nitrogen. In the case of nitrogen the maximum deviation from the mean value of $(\epsilon-1)v$ is less than one percent which was the upper limit estimated for the error introduced by the original calibration of the condensers used.[2]

Measurements were also made on $CO_2$, $NH_3$, and $SO_2$. The $CO_2$ and $SO_2$ were taken from commercial cylinders and dried and then distilled repeatedly by freezing in liquid air and evaporating, discarding each time small quantities of the gas coming off first and last. $NH_3$ was obtained by heating concentrated ammonia water and was purified as above.

The measurements are given in Table I and the values of $(\epsilon-1)vT$ are plotted against $T$ in Fig. 1. All these measurements were made by measuring the difference in the capacities of the experimental condenser corresponding to atmospheric pressure and a small pressure of about

[3] P. Debye, Phys. Zeits. **13**, 97 (1912).

6 cm of the gas under investigation. (This procedure was adopted because it was found that when the condenser was completely evacuated the heat insulation between the plates was good enough to prevent sufficiently rapid equalization of temperature of the two plates.) The results are reduced to atmospheric pressure by assuming that $(\epsilon-1)$ may be regarded as proportional to the pressure for this small correction. In each case, measurements were made to temperatures within a few degrees of the liquefaction points corresponding to atmospheric pressure. These later points are indicated in the figure by vertical broken lines.

TABLE I

*Dielectric constant data*

| $T°K$ | $(\epsilon-1)\times10^6$ | $(\epsilon-1)v\times10^6$ | $(\epsilon-1)vT$ |
|---|---|---|---|
| | Nitrogen | | |
| 84.1 | 1898 | 572 | .0481 |
| 197.8 | 792 | 572 | .1132 |
| 273.0 | 581 | 581 | .1586 |
| 562.1 | 283 | 583 | .3276 |
| | Carbon Dioxide | | |
| 199.2 | 1353 | 978 | .1948 |
| 200.6 | 1345 | 978 | .1962 |
| 286.6 | 926 | 968 | .2774 |
| 295.0 | 909 | 976 | .2878 |
| 469.0 | 561 | 962 | .4512 |
| | Ammonia | | |
| 241.7 | 9280 | 8142 | 1.966 |
| 274.4 | 7132 | 7143 | 1.962 |
| 296.7 | 6117 | 6620 | 1.965 |
| 371.4 | 4018 | 5452 | 2.024 |
| 456.9 | 2734 | 4567 | 2.086 |
| | Sulfur Dioxide | | |
| 265.5 | 10015 | 9627 | 2.547 |
| 267.6 | 9918 | 9530 | 2.550 |
| 295.5 | 8176 | 8770 | 2.592 |
| 297.2 | 8120 | 8768 | 2.606 |
| 366.9 | 5477 | 7323 | 2.687 |
| 443.8 | 3911 | 6336 | 2.812 |

The data of particular interest are those for $SO_2$ and $NH_3$. For $SO_2$ Jona obtained at 300°F about a 10% increase of $(\epsilon-1)vT$ over the value corresponding to the limiting slope at higher temperatures; whereas the observations here recorded show no such anomaly, even though the measurements are carried about 35° below Jona's lowest temperature. For $NH_3$ Jona recorded at room temperature a slight anomaly, whereas Fig. 1 shows no definite deviation for temperatures as low as room temperature, but near the liquefaction point there seems to be a definite increase of about 2%. The obvious inference is that there is some effect that is present only slightly in the present measurements but which was

quite appreciable in Jona's work. It is difficult to imagine how such an effect could be attributed to association which should reach a definite equilibrium concentration, unless the time required for association should be comparable to the time between observations. This latter supposition seems very unlikely. In fact it is difficult to see how any effect which is characteristic of the gas itself; i.e., any volume effect could account for the difference in the two sets of observations, whereas the existence of surface layers on the metallic plates might reasonably be responsible for the inconsistencies. Such layers might depend upon the nature of the

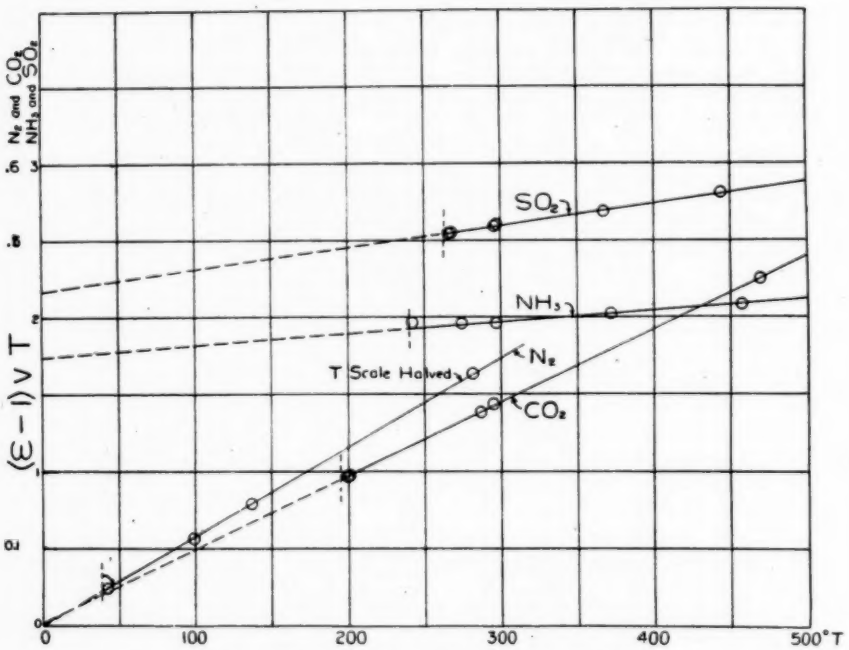


Fig. 1. Variation of $(\epsilon-1)vT$ with temperature.

metal of the condenser and also upon the degree to which the surfaces had been cleaned by baking and evacuation. For example it is possible that a surface which had not been properly cleaned might have entirely different properties as regards adsorption of layers of the gas. In fact some recent experiments of the author on water vapour suggest that there is a surface effect in such measurements.[4]

In Table II are given the Debye constants as taken from the data of Fig. 1 together with the values of electric moment given by Jona, for the purpose of comparison. From the figure it would appear that $CO_2$ has a

[4] C. T. Zahn, Phys. Rev. **27**, 329 (March, 1926).

slight electric moment but considerably less than that given by Jona. In fact the value is so small as to be within the limit of experimental error, and it cannot be stated definitely that $CO_2$ has an appreciable electric moment. The difference between Jona's values of $\mu$ and those taken from Fig. 1 may be due to an error introduced by his equation

TABLE II

*The Debye constants*

| Gas | $A\times10^6$ | $B$ | $\mu\times10^{18}$ (c.g.s.,e.s. units) | $\mu\times10^{18}$(Jona) |
|---|---|---|---|---|
| $CO_2$ | 958 | .003 | .06 | .303 |
| $NH_3$ | 768 | 1.740 | 1.44 | 1.53 |
| $SO_2$ | 1433 | 2.167 | 1.611 | 1.76 |

which was developed on the assumption of the existence of associated molecules. This error would present itself in the reduction to standard pressure and would depend on the actual pressures used in the experiments. Since he has not given these pressures, no definite comparison can be made. The difference in the $\mu$ values of Table II is about 7% for $NH_3$ and $SO_2$ and is well beyond the limit of experimental error for both observers.

In Table III is given a resumé of all the observations taken of the variation of dielectric constant with temperatures with the apparatus

TABLE III

*Resumé of dielectric constant data*

| Gas | $A\times10^6$ | $B$ | $\mu\times10^{18}$ (c.g.s.,e.s. units) |
|---|---|---|---|
| HCl | 1040 | .895 | 1.03 |
| HBr | 1212 | .52 | .79 |
| HI | 1856 | .12 | .38 |
| $C_2H_2$ | 1334 | 0 | 0 |
| $C_2H_4$ | 1444 | 0 | 0 |
| $C_2H_6$ | 1501 | 0 | 0 |
| $\alpha$–$C_4H_8$ | 2694 | .115 | .37 |
| $N_2$ | 580 | 0 | 0 |
| $CO_2$ | 958 | .003 | .06 |
| $NH_3$ | 768 | 1.74 | 1.44 |
| $SO_2$ | 1433 | 2.167 | 1.61 |

described previously by the author.[2] There are here included the measurements on the halogen hydrides, those here described, and measurements taken on four hydrocarbons in collaboration with Dr. Charles P. Smyth.[5]

In conclusion the author wishes to acknowledge his indebtedness to Professor K. T. Compton for helpful advice and to Mr. E. W. Cheney for assistance in making some of the measurements.

NATIONAL RESEARCH FELLOWSHIP.
PRINCETON UNIVERSITY.
November, 1925.

[5] C. P. Smyth and C. T. Zahn, Jour. Amer. Chem. Soc. **47**, 2501 (1925).

# FORCES ON A RIGID MAGNETIZED CONDUCTOR

By E. H. Kennard and S. C. Wang

Abstract

The *forces on magnetized material carrying current* are examined from the standpoint of the electron theory. The true distribution of forces depends upon the minute structure of the material and so is at present unknown in the general case; it is even doubtful whether the partial force on the current alone is $\mathbf{j}\times\mathbf{B}$. For a *rigid body* two sets of forces equivalent to the unknown actual ones are obtained: the first set consists of a force per unit volume equal to $\nabla_H(\mathbf{I}\cdot\mathbf{H})+\mathbf{j}\times\mathbf{B}$ together with an additional torque per unit volume $\mathbf{I}\times\mathbf{H}$; the second consists simply of a force per unit volume equal to $m\mathbf{H}+\mathbf{j}\times\mathbf{H}$, ($\mathbf{H}$ = magnetic intensity, $\mathbf{B}$ = induction, $\mathbf{j}$ = current density, $\mathbf{I}$ = intensity of magnetization, $m$ = free pole density). The first set may resemble the actual forces somewhat more closely than the second, but the second set is so much simpler that it is recommended for general adoption. If a set of stresses is preferred to body forces, the set commonly used is recommended, which consists of a tension along the tubes and a pressure perpendicular to them, each equal to $\mu H^2/8\pi$, with no assumption whatever as to the stresses in rigid bodies where $\mu$ does not have a constant value: this set corresponds to neither of the laws of force.

The *rotating-magnet experiment* recently repeated by Zeleny and Page is discussed and the torque is ascribed by preference to forces on the free poles.

A PROBLEM in electromagnetics that is still unsettled is that of the forces that act upon a permanently magnetized conductor carrying a current. The source of difficulty here lies in the fact that permanent magnetism occurs only in solids, and in the case of a solid body only the total force and torque upon it are accessible to experiment and not the detailed distribution of force throughout the body. The question is further complicated by the attempt one naturally makes to separate the force into two parts, a force upon the current, and a force upon the magnetic material. Since these two partial forces cannot possibly be separated in observation, such a separation can be effected only upon the basis of theoretical assumptions of some sort.

In such a situation the theorist often tries first a simple extension to the new case of laws that are well established for simpler cases. In the present case, however, such an attempt meets with serious difficulty. For one thing, the force upon a current can be observed separately only in media where magnetic intensity and induction are equal; when we pass to magnetic material, we are in doubt which of these vectors, or what combination of them, should be taken to determine the partial force upon the current. In the second place, the field in a current-containing region differs notably from that due to magnets in that it possesses no scalar

potential. The forces exerted by such a field upon a small plane current are not the same as the forces upon the "equivalent" magnetic dipole, as is easily seen in certain simple cases. What, then, shall we assume to be the action of such a field upon a piece of magnetized matter placed in the field? No experimental answer to this question is possible, for the simple reason that such fields can only be realized in the presence of distributed currents and the force is then masked by the unknown partial force upon the current.

Under these circumstances a definite answer seems to be possible only on the basis of a constitutive theory of magnetism and current. Today the safest basis appears to be the electron theory, and it is the purpose of the present paper to treat the problem systematically from this point of view; no adequate treatment on this basis seems to occur in the literature. All that we shall need to borrow from the electron theory is the assumption that magnetic fields and electromagnetic forces are due to moving electricity acting according to the laws of Lorentz; for simplicity we shall speak in terms of electrons, but our discussion will have nothing to do with the attempts, hitherto largely unsuccessful, to account for the specific magnetic or electric properties of particular kinds of matter. The results of the investigation are carefully summarized in the Abstract above.

The problem is made particularly timely by the recent interesting repetition by Zeleny and Page[1] of the experiment in which a bar magnet is set into rotation about its magnetic axis by allowing a current to enter it in the middle and leave at one end. The mode in which the torque to be expected is calculated by these authors, although simple, does not seem to us to be, as it stands, free from logical objection. Our principal purpose here is not to criticize this calculation, but we think it may assist in clarifying the subject if we first explain our objections to it.

*The rotating magnet as treated by Zeleny and Page.* In treating the theory of the rotating magnet, the authors imagine the current to enter in the form of a symmetrical "sheet" $AB$; and they suppose that either the magnetization or the distribution of current in the sheet is symmetrical about the axis. They then calculate the torque in two steps. The torque "exerted by the magnet upon the current" is calculated from the assumption that this force is proportional to the induction; the basis of this assumption is presumably the electron theory. The "torque exerted by the current on the magnet" is then calculated from the forces on the free poles, and a novel procedure is here introduced. Taking first the simple case in which the current enters in a thin sheet $AB$, it is

[1] Zeleny and Page, Phys. Rev. **24**, 544 (1924).

pointed out that there is a distribution of free poles *CC* on the material just above the sheet which is equal and opposite to that on the end of the magnet at *B*, and obviously the torques on these two sets of poles will exactly cancel each other; the net torque on the magnetic material is therefore concluded to be zero. The *total* torque as thus calculated was found to agree with the experimental value.

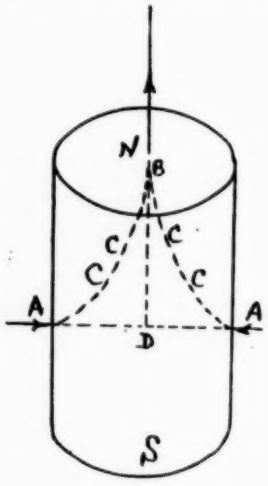


Fig. 1.

An important point would seem, however, to have been overlooked. Presumably the current is supposed to flow through a thin but finite slice of the material; this slice will then have poles, and the torque on the upper poles of the slice will clearly be just equal and opposite to the torque on the contiguous poles *CC* just above the current. If account is taken of this torque on the slice the total torque on the magnetic material no longer vanishes. Probably the writers of the article knowingly rejected the torque on the slice as negligible, on the excellent and generally valid principle that forces on indefinitely small things must likewise be indefinitely small. But the present case is peculiar in that the current density in the slice is very high and some of the derivatives of the magnetic field, upon which the torque must depend, are very large; if the slice is made indefinitely thin these quantities become indefinitely large (at least relatively). For this reason it appears to the present writers that an additional argument is necessary at this point.

Apparently the difficulty can be met only by invoking some new law of force for application to the current-carrying slice. Perhaps with an emendation of this sort the calculation would appeal to some as satisfactory. In the opinion of the present writers, however, the simultaneous use of two different laws of force in dealing with a single piece of matter seems complicated and confusing and likely to lead to errors. This objection seems to gain in weight when we turn to the actual case in which no part of the magnet is entirely free from current.

In the experiments referred to, the magnet was also replaced by a brass cylinder and it was observed experimentally that this cylinder experienced the same torque for a given current as did the magnet provided the magnetic flux was the same in the two cases; this result was taken to support the view that the "poles of a symmetrical magnet

contribute nothing to the torque" under the circumstances of these experiments. It seems obvious, however, that such a conclusion can be drawn only after we have shown that the partial torque on the current is the same in the two cases, in spite of the difference that in one case the flux is due entirely to field-strength whereas in the other case it is largely flux of magnetization; and since this cannot be shown experimentally, we are thrown back upon theory again, so that the results with the brass cylinder hardly seem to throw much light upon the problem.

The true nature of these difficulties as we see them will become clearer after our general discussion of the law of force, to which we now turn.

## FORCES AND TORQUES ON A RIGID MAGNETIC BODY TRAVERSED BY CURRENTS

*Nomenclature used*:

$\mathbf{B}$ = the magnetic induction.

$\mathbf{H}$ = the magnetic intensity.

$\mathbf{H}'$ = the partial magnetic intensity at a point in the magnetic material obtained by omitting the magnetic field of the entire body.

$\mathbf{I}$ = the magnetic moment per unit volume or the intensity of magnetization.

$m = -\operatorname{div}\mathbf{I}$ = the free pole density.

$\mathbf{j}$ = the current density.

*First form of the law of force.* We shall begin our calculation with the simple assumption that the electron streams constituting the current go through the material in straight paths on the molecular scale. In such a case, since the average value of the true magnetic intensity along these paths is, according to the electron theory, the magnetic induction $\mathbf{B}$, the average force on the moving electricity per unit volume is evidently $\mathbf{j}\times\mathbf{B}$.

In calculating the force on the molecular circuits of the magnetic body, we shall make use of the following general principle: The work done by electromagnetic forces on the molecular circuits of a piece of rigid magnetic matter, when the latter is virtually displaced in any manner, is $\int \delta_{H'}(\mathbf{I}\cdot\mathbf{H}')d\tau$ integrated over the piece of the material, where $(\mathbf{I}\cdot\mathbf{H}')$ denotes the scalar product of the two vectors, $\delta_{H'}(\mathbf{I}\cdot\mathbf{H}')$ is the change of $(\mathbf{I}\cdot\mathbf{H}')$ at a point of the material after the displacement and the subscript $H'$ indicates that the intensity of magnetization $\mathbf{I}$ is to be fixed in magnitude and direction relative to the material in calculating the change.

According to this principle the $x$, $y$, and $z$ components of the force due to $\mathbf{H}'$ on the molecular circuits of the material per unit volume are

$\mathbf{I}\cdot\partial\mathbf{H}'/\partial x$, $\mathbf{I}\cdot\partial\mathbf{H}'/\partial y$ and $\mathbf{I}\cdot\partial\mathbf{H}'/\partial z$ respectively; or the vector force $\mathbf{f}'$ per unit volume is $\nabla_{H'}(\mathbf{I}\cdot\mathbf{H}')$, where the subscript $H'$ denotes again that the differentiation is to be made on $\mathbf{H}'$ alone.

Adding to $\mathbf{f}'$ the force on moving electricity, $\mathbf{j}\times\mathbf{B}$, we get for the total translational force per unit volume on the material due to $\mathbf{H}'$

$$\mathbf{F}'=\nabla_{H'}(\mathbf{I}\cdot\mathbf{H}')+\mathbf{j}\times\mathbf{B}\ . \tag{1a}$$

It will be shown later that in the above equation $\mathbf{H}'$ can be replaced by $\mathbf{H}$ without affecting the value of the total force on the rigid body. Hence we can also employ instead of $\mathbf{F}'$ a force $\mathbf{F}_1$ per unit volume given by[1]

$$\mathbf{F}_1=\nabla_H(\mathbf{I}\cdot\mathbf{H})+\mathbf{j}\times\mathbf{B}\ . \tag{1}$$

The field will also exert a torque on the magnetic matter tending to rotate the molecular circuits about an axis perpendicular to the plane of the vectors $\mathbf{I}$ and $\mathbf{H}'$. If $\theta$ is the angle between $\mathbf{I}$ and $\mathbf{H}'$ the magnitude of this torque will be $\partial/\partial\theta(\mathbf{I}\cdot\mathbf{H}')=-IH'\sin\theta$, so that the torque per unit volume exerted independently of the translational force $\mathbf{F}'$ will be

$$\mathbf{T}'=\mathbf{I}\times\mathbf{H}'\ ; \tag{2a}$$

or, replacing $\mathbf{H}'$ by $\mathbf{H}$ for the reason just stated, we can employ as the independent torque per unit volume

$$\mathbf{T}=\mathbf{I}\times\mathbf{H}\ . \tag{2}$$

The total vector torque on the body about the origin will thus be

$$\mathbf{L}=\int[\mathbf{r}\times\mathbf{F}_1+\mathbf{I}\times\mathbf{H}]\,d\tau=\int\{\mathbf{r}\times[\nabla_H(\mathbf{I}\cdot\mathbf{H})+\mathbf{j}\times\mathbf{B}]+\mathbf{I}\times\mathbf{H}\}\,d\tau\ , \tag{3}$$

where $\mathbf{r}$ is the radius vector from the origin to the element $d\tau$.

We shall now drop the assumption that the conduction electrons go through the material in straight paths on the molecular scale. Since the reaction of the force on the body, as exerted on the rest of the system, will depend only upon dimensions or properties of that body on the physical scale, it is evident that the total force on the rigid body will remain the same as long as the paths of the electrons of two similar systems do not differ on the physical scale. Hence Eqs. (1a) to (3) will certainly lead to correct results for an actual rigid body.

We cannot, however, attach with certainty any definite physical meaning to the two terms of Eq. (1) or (1a) separately. To say that the one

[1] This force is the same as that given in Livens' Theory of Electricity, p. 589, but the torque (2a) is not there given. If Curl $\mathbf{H}=0$, $\nabla_H(\mathbf{I}\cdot\mathbf{H})=(\mathbf{I}\cdot\nabla)\mathbf{H}$ and Eq. (1) and (2) agree with the forces on p. 270 of Livens' book.

is the force on the current and the other that on the magnet would be justified only if the paths of the electrons were known to be straight lines on the molecular scale or if we could ascertain by some other means that the average force on moving electricity really is $\mathbf{j}\times\mathbf{B}$ per unit volume.

These equations cannot even be regarded as giving the true total force-action per unit volume upon the material, for there will probably be additional forces and torques which arise from local attractions and repulsions between the molecular circuits and the moving electrons and which are not included in the force-action as calculated from such mean values as the ordinary current density and magnetization. The probability of such effects is easily seen by considering special cases. This local action will not affect the motion of the body as a whole, of course, but it may alter the distribution of the forces and so may give rise to strains; for this reason, as is well known, magnetostrictive effects cannot be calculated from equations such as (1) to (3).

*Second form of the law of force.* From Eq. (1a) the $x$-component of the total force on the body is

$$P_x=\int\left[\mathbf{I}\cdot\frac{\partial\mathbf{H}'}{\partial x}+(\mathbf{j}\times\mathbf{B})_x\right]d\tau \tag{4}$$

$$=\int\left[I_x\frac{\partial H_x'}{\partial x}+I_y\left(\frac{\partial H_y'}{\partial x}-\frac{\partial H_x'}{\partial y}\right)+I_y\frac{\partial H_z'}{\partial y}-I_z\left(\frac{\partial H_z'}{\partial z}-\frac{\partial H_z'}{\partial x}\right)\right.$$

$$\left.+I_z\frac{\partial H_z'}{\partial z}+(\mathbf{j}\times\mathbf{B})_x\right]d\tau$$

$$=\int\left[(\mathbf{I}\times\operatorname{Curl}\mathbf{H}')_x+\frac{\partial}{\partial x}(I_xH_x')+\frac{\partial}{\partial y}(I_yH_x')+\frac{\partial}{\partial z}(I_zH_x')\right.$$

$$\left.-H_x'\frac{\partial I_x}{\partial x}-H_x'\frac{\partial I_y}{\partial x}-H_x'\frac{\partial I_z}{\partial z}+(\mathbf{j}\times\mathbf{B})_x\right]d\tau$$

$$=\int[(\mathbf{I}\times\operatorname{Curl}\mathbf{H}')_x+\operatorname{div}(H_x'\mathbf{I})-H_x'\operatorname{div}\mathbf{I}+(\mathbf{j}\times\mathbf{B})_x]d\tau\,.$$

Substituting $\operatorname{Curl}\mathbf{H}'=4\pi\mathbf{j}$, $\mathbf{B}-4\pi\mathbf{I}=\mathbf{H}$ and $-\operatorname{div}\mathbf{I}=m$, we have

$$P_x=\int[(\mathbf{j}\times\mathbf{H})_x+mH_x'+\operatorname{div}(H_x'\mathbf{I})]d\tau\,. \tag{5}$$

By the divergence theorem $\int\operatorname{div}(H_x'\mathbf{I})d\tau$ integrated over the body is equal to $\int H_x'I_n dS$ integrated over a surface immediately surrounding

the body. But this surface integral vanishes because $\mathbf{I}$ vanishes on the surface. Eq. (5) then becomes

$$P_x = \int [mH_x' + (\mathbf{j}\times\mathbf{H})_x]d\tau \ ; \tag{6}$$

and similar results are obtained for $P_y$ and $P_z$.

Turning now to the total torque, we have from Eqs. (1a) and (2a) for its $z$-component

$$L_z = \int \left\{ x\left[\mathbf{I}\cdot\frac{\partial \mathbf{H}'}{\partial y} + (\mathbf{j}\times\mathbf{B})_y\right] - y\left[\mathbf{I}\cdot\frac{\partial \mathbf{H}'}{\partial x} + (\mathbf{j}\times\mathbf{B})_x\right] + (\mathbf{I}\times\mathbf{H}')_z \right\} d\tau \tag{7}$$

Here we can operate on the terms inside the two brackets exactly in the same manner as we did in passing from Eq. (4) to Eq. (5). Corresponding to Eq. (5), Eq. (7) becomes

$$\begin{aligned} L_z &= \int \left\{ x[(\mathbf{j}\times\mathbf{H})_y + mH_y' + \operatorname{div}(H_y'\mathbf{I})] - y[(\mathbf{j}\times\mathbf{H})_x + mH_x' \right. \\ &\qquad \left. + \operatorname{div}(H_x'\mathbf{I})] + (\mathbf{I}\times\mathbf{H}')_z \right\} d\tau \\ &= \int \left\{ x[(\mathbf{j}\times\mathbf{H})_y + mH_y'] - y[(\mathbf{j}\times\mathbf{H})_x + mH_x'] + \operatorname{div}(xH_y'\mathbf{I}) \right. \\ &\qquad \left. - H_y' I_x - \operatorname{div}(yH_x'\mathbf{I}) + H_x' I_y + (\mathbf{I}\times\mathbf{H}')_z \right\} d\tau . \end{aligned}$$

By the divergence theorem the volume integrals of the two divergence terms are equal to two surface integrals which vanish. Hence we have

$$L_z = \int \left\{ x[(\mathbf{j}\times\mathbf{H})_y + mH_y'] - y[(\mathbf{j}\times\mathbf{H})_x + mH_x'] \right\} d\tau \ , \tag{8}$$

with similar expressions for $L_x$ and $L_y$.

According to Eqs. (6) and (8) the resultant force and torque are the same as if the body experienced a force per unit volume equal to $m\mathbf{H}' + \mathbf{j}\times\mathbf{H}$. To this force we can, however, add a force $m(\mathbf{H}-\mathbf{H}')$ without affecting the total force and torque, for the forces thereby added are simply the forces on the free poles of the body due to their own fields. Hence the total force and torque on the body are the same as if the only force acting were a force per unit volume given by

$$\mathbf{F}_2 = m\mathbf{H} + \mathbf{j}\times\mathbf{H} \ . \tag{9}$$

It is evident that if we replace $\mathbf{H}'$ by $\mathbf{H}$ in Eqs. (6) and (8) and trace back step by step, we will finally get Eq. (1) instead of (1a) and (2) instead of (2a), as already stated in connection with these equations.

*Discussion of the two forms of the law.* We now have two simple laws of force between which we can choose. The first form of the law[2] is represented by Eqs. (1) and (2); in this form there is a force $\mathbf{j}\times\mathbf{B}$ on the current, and then there are a force $\nabla_H(\mathbf{I}\cdot\mathbf{H})$ and a torque $\mathbf{I}\times\mathbf{H}$ which together represent a tendency of the magnetic material to move so as to increase the flux of magnetic intensity in the direction of the magnetization. This form of the law has an advantage in that it almost certainly resembles the true distribution of forces more closely than does the other distribution given by Eq. (9), without, however, any strong claim to be correct in detail; it has been pointed out before that in reality neither the separate terms of $\mathbf{F}_1$ nor $\mathbf{F}_1$ as a whole can be regarded as representing with certainty the true distribution of forces upon the material. This form has also the advantage of accounting directly for effects not only upon rigid bodies but upon incompressible liquids as well (one has only to deduce from this law the body forces represented, for instance, by the middle term of Eq. (407) in Jeans' Electricity and Magnetism, 4th ed.).

On the other hand, the second form of the law of force as given in Eq. (9) has the great merit of simplicity.[3] It involves only a force, without the addition of a torque, and the expression for the force is especially simple, since both parts of it involve the same field vector; the force consists of a force $m\mathbf{H}$ upon the free poles, which are so useful in calculating magnetic fields, and a force $\mathbf{j}\times\mathbf{H}$ upon the current. Furthermore, in case of need the second form can be easily made to give a distribution of fo ces exactly like that given by the first form by applying it, not to the free poles of the whole body, but to the poles of the elements; the first term in the force equation then gives $\nabla_H(\mathbf{I}\cdot\mathbf{H})-\mathbf{I}\times\mathrm{Curl}\,\mathbf{H}$ and thus compensates for the difference between $\mathbf{j}\times\mathbf{H}$ and $\mathbf{j}\times\mathbf{B}$. Such a procedure seems very natural in the case of a liquid, whose elements are capable of relative motion; thus this case also can be handled without difficulty by means of the second form of the law, and the calculation is found to be little longer than if the first form is employed.

After weighing these considerations carefully, the present authors would recommend the retention or adoption in elementary treatises of the second law of force, Eq. (9), as the basic form. In advanced treatises both laws may, of course, be given.

[2] The law is given in this form in Livens' Theory of Electricity, pp. 270, 589.

[3] This form is equivalent to the law stated by A. Einstein and J. Laub (Annalen der Phys. **26,** p. 541, 1908, Eqs. (7) and (8)); the only difference is that they represent the first part of the force as acting upon the poles of the elements instead of the free poles.

In any case, it would seem to be best not to mix the two laws in treating the same rigid body. For instance, in the paper of Zeleny and Page previously referred to, the force $m\mathbf{H}$ on the poles as required by the second law is combined with the force on the current in the form $\mathbf{j}\times\mathbf{B}$ as required by the first law; this undesirable combination was, in our opinion, the source of the difficulties discussed above.

To avoid misunderstanding, a word should be said about the force on a wire surrounded by a magnetic fluid, which equals $\mathbf{j}\times\mathbf{B}_0$, where $\mathbf{B}_0$ is the value that $\mathbf{B}$ would have at a point on the wire if a short section of it and if its current were removed. Neither of the laws deduced above gives a force of this magnitude as acting upon the wire (consider, for instance, the case in which the wire itself is non-magnetic). The deficiency is made up by a purely mechanical differential pressure of the fluid upon the wire; the existence of such a pressure would become evident if the wire were surrounded by a narrow empty space and a sheath carrying no current, for this part of the force would then act upon the sheath. Thus the fact that the total force on the wire is $\mathbf{j}\times\mathbf{B}_0$ does not lend support, as might be thought, to what we have called the first form of the law.

In conclusion we shall compare briefly the treatment, in terms of the two laws, of the case referred to above in which a current enters a cylindrical bar magnet along a circle $AA$ (see Fig. 1) round its middle and leaves at one end along the axis of rotation. The calculation can be put into a very simple form if we first note that the torque is necessarily independent of the path taken by the current through the magnet, because the difference in distribution between any two paths consists of a distribution of current confined to the interior and surface of the magnet, and interaction between such a distribution and the magnet cannot produce a net torque on the magnet itself. We can therefore suppose the current to enter symmetrically around the ring $AA$, flow radially to the center, and then follow the axis to the end.

If we now use the first law of force, we see at once from symmetry that the torque in Eq. (2) and the first term in Eq. (1) contribute nothing to the torque. A simple calculation then shows that the second term of (1), $\mathbf{j}\times\mathbf{B}$, gives a torque equal to $iN$, where $i$ = total current, $N$ = flux of induction across the section $AA$ of the magnet.

If we use, instead, the second law, represented by Eq. (9), we find by a simple calculation that the second term, $\mathbf{j}\times\mathbf{H}$, gives a torque $iN_H$ where $N_H$ = flux of intensity $H$ through the section $AA$; this torque is small if the magnet is very slender. The first term is then easily seen to yield a torque on the poles equal to $i(\Sigma m)/4\pi = iN_1$, where $N_1$ flux of magnetization through $AA$. Since $N_H + N_1 = N$, the total torque is $iN$.

It is clear that in this special case the first law of force leads more simply to the final result. It appears, however, to the present writers that this case is a peculiar and atypical one, and that in spite of the disadvantage in this case the second form of the law is nevertheless the best one to choose whenever it is desired to state a single universal law applicable to all cases.

The peculiar symmetry of this case also enables us to go a little further than usual in regard to the true location of the forces: if both the magnetization and the current are symmetrically distributed around the axis, then it seems clear that local forces can have no component perpendicular to a plane through the axis so that the torque-producing component of force at each point must really be the corresponding component of $\mathbf{j}\times\mathbf{B}$; but the same statement cannot be made of other components of the force, nor can it be asserted with confidence that $\mathbf{j}\times\mathbf{B}$ represents the force acting directly upon the current.

*Stress-equivalents for the law of force.* The set of forces given by Eq. (9) is identical with that of Eq. (20) on page 282 of the third edition of Maxwell's Electricity and Magnetism. It is shown there that this set of forces is equivalent to the following set of stresses:

(a) a tension along the lines of force equal to $H^2/8\pi$,

(b) a pressure in all directions at right angles to the lines of force numerically equal also to $H^2/8\pi$.

Many other sets of stresses can be assigned which will account correctly for the force action upon a rigid body, for this depends only upon the stresses over its surface. The choice of a set will be dictated purely by convenience, since the occurrence of local action precludes any attempt to calculate the strains of magnetostriction in the general case. In practice, the stresses are commonly taken to be of the magnitude $\mu H^2/8\pi$ at points where there is a constant scalar permeability; this expression has the well-known advantage of accounting correctly for the behavior not only of rigid bodies but also of incompressible isotropic liquids. The most practical plan would seem to be to continue this practice and simply to assume nothing in regard to the stresses in material where such a permeability does not exist, for the latter happens only in rigid bodies where the magnitude of the stresses is immaterial anyhow.

It is interesting to note that the body forces due to the stresses of magnitude $\mu H^2/8\pi$ correspond to neither of the two laws of force developed above.

CORNELL UNIVERSITY,
Nov. 17, 1925.

# NOTE ON "FORCES ON A RIGID MAGNETIZED CONDUCTOR"

By John Zeleny and Leigh Page

Abstract

In reply to the criticism by Kennard and Wang of our theory of the torque on a cylindrical magnet through which a current is passing, it is pointed out that there is a rational distinction between the torque exerted by the current on the magnet and that exerted by the magnet on the current passing through it. From the point of view of Ampere's theory of magnetism, the first torque is zero and the second is $iN/2\pi$, in electromagnetic units, both for the case of a symmetrical current sheet and asymmetrical magnetization and for the case of symmetrical magnetization and an asymmetrical current. Therefore on the modern conception that magnetic properties are due to electric currents and not to magnetic charges *per se*, the torque on a cylindrical magnet carrying a current cannot correctly be ascribed to the action of the current on the magnet in either of the two cases under discussion, contrary to the contention of Kennard and Wang.

IN a recent paper[1] we deduced a new formula for the torque on a cylindrical magnet through which a current is passing in terms of the flux $N$ and the current $i$ and we verified this formula experimentally over a considerable range. In the paper preceding this note Kennard and Wang criticize our theory on the grounds (a) that it is impossible to make any rational distinction between the torque exerted by the current on the magnet and that exerted by the magnet on the current passing through it, (b) that the point of view adopted by us in calculating the first torque is inconsistent with that used in calculating the second and that we obtained the right result only by a procedure of dubious legitimacy, (c) that it is preferable to ascribe the torque to forces on the free poles rather than to forces on the current passing through the magnet.

At the start we wish to make clear that our point of view is that of Ampère's theory of magnetism, that is, that the magnetic properties of a magnet are due to intra-molecular or intra-atomic currents of constant intensity and not to magnetic charges *per se*. The question as to whether these currents are due to revolution of electrons about a nucleus or not is irrelevant to the analysis of the phenomenon. To be sure, we found it convenient at one point of our argument to substitute for the Amperian circuits a fictitious density of magnetism in accord with the usual procedure. This device, however, did not involve a change in point of view.

[1] Zeleny and Page, Phys. Rev. **24**, 544 (1924).

In answer to objection (a) we observe that the field $H$ due to the current in the magnet as well as that outside is a perfectly definite quantity which is calculable in the same manner whether the current passes through an iron magnet or a brass cylinder. Its value depends only on the value $i$ of the steady current and the geometrical configuration of the circuit. By the torque exerted by the current on the magnet we mean the torque exerted by this field on the magnet. Similarly the field due to the Amperian circuits responsible for the magnetic properties of the iron is a perfectly well defined quantity. The average value of this field, as is well known, is the magnetic induction $B$. By the torque exerted by the magnet on the current passing through it we mean the torque exerted by this field on the current. So we maintain that on the Amperian theory of magnetism (which we believe is generally accepted by physicists)there is a rational distinction between the two torques calculated in our paper.

Objections (b) and (c) are answered by a consideration of the calculation of the two torques defined above. The force on a current element is given by the force equation of the Maxwell-Lorentz theory. Substituting $B$ for the magnetic intensity in this equation, the torque $iN/2\pi$ exerted by the magnet on the current is obtained, as given in our original paper. To be sure, since $B$ is the average value of the field inside the magnet, the natural assumption is made that the electrons carrying the current do not show any preference for the weaker or stronger portion of the field in their passage through the material. *The agreement of our experimental results with the theory, certainly in the case of a symmetrical current sheet, is an a posteriori justification of this assumption.* Moreover, we fail to see that Kennard and Wang's analysis of the problem avoids this assumption. In calculating the torque exerted by the current on the magnet we made use of a fictitious density of pole, and showed that, for the two cases considered, there is no torque due to the action of the current on the magnet. Kennard and Wang seem to consider that this substitution of a fictitious density of magnetism for the Amperian molecular currents actually present involves a change in point of view and they make a point of the possibility of the current's passing between the two surfaces of such a fictitious magnetic shell. Now it is well known that the field of a current circuit is equivalent to that of a magnetic shell, whose periphery coincides with the circuit, only at points outside the shell. Therefore when considering the interaction of the current and the magnet the fictitious magnetic shells introduced must be so placed that the current under consideration is never located in the interior of a shell. This is always possible since the only condition determining the form and location of the shell which has to be satisfied is that its periphery shall

coincide with the circuit. However, it is a simple matter to show that the torque exerted by the current on the magnet vanishes without introducing a fictitious magnetic charge. The proof is given below; the two cases considered in the original paper being treated separately.

*Case I. The distribution of current in the sheet is symmetrical.* In this case the lines of force of the field due to the current are circles in planes perpendicular to the axis of the magnet with centers on this axis. As the force on any current element at a point $P$ in the magnet is perpendicular to the field at this point it lies in the plane determined by the axis and the point $P$. Hence it has no moment about the axis of the magnet. Therefore there is no torque exerted by the field of the current on the Amperian circuits in the magnet, no matter how asymmetric the magnetization of the magnet may be.

*Case II. The magnet is symmetrically magnetized.* Take an annular ring of radius $p$ and cross section $ds$ about the axis of the magnet. Use cylindrical coordinates $p$, $\phi$, $z$, the last named being measured parallel to the axis of the magnet. Let $J_p$, $J_\phi$, $J_z$ be the sums of the components of all the current elements of the Amperian circuits per unit volume in the directions of increasing $p$, $\phi$, $z$ respectively, *the current elements of the current $i$ being excluded from these sums.* Denote by $H_p$, $H_\phi$, $H_z$ the components of the field due to $i$ resolved in these same directions and by $dF_p$, $dF_\phi$, $dF_z$ the components of the force exerted by $H$ on the current elements comprised in $J$ which are contained in the volume element $pd\phi ds$. Then the only component of $dF$ which has a moment about the axis is $dF_\phi$. Now

$$dF_\phi = (-J_p H_z + J_z H_p)\, pd\phi ds$$

But in a symmetrically magnetized bar $J_p = J_z = 0$. Therefore the force $F_\phi$ vanishes, whatever $H_z$ and $H_p$ may be, that is, whatever form the current circuit $i$ may have. It should be noted that symmetrical magnetization does not mean that $J_\phi = 0$ throughout the magnet, and it is only because $J_\phi$ does not enter the expression for $dF_\phi$ that we can conclude that $dF_\phi$ is zero. For instance, if the magnet is magnetized uniformly as well as symmetrically (intensity of magnetization constant and everywhere parallel to the axis) $J_\phi$ is zero in the interior of the magnet but not at the cylindrical surface. In this case the resultant effect is that of a solenoid wound around the magnet. Incidentally it may be recalled that in our experiments symmetry of magnetization was tested by approaching a second magnet to the suspended magnet and noting whether or not a deflection was produced.

The only possible torques which have been neglected are (a) a torque on the magnet due to the field of the magnet, (b) a torque on the current due to the field of the current. Obviously there can be no torque (a). As regards (b) the treatment of case I above shows that there can be no torque of this character when the distribution of current in the sheet is symmetrical. When the distribution of current is not symmetrical the torque (b) on the portion of the current in the rotating system may not vanish and cases can be imagined in which it might be of appreciable magnitude, but it is obviously quite negligible in the usual set up such as was employed in the experiments described in our original paper.

Finally, the rational distinction which we have made between torque on the current and torque on the magnet vindicates our claim that the experiments with a brass cylinder placed in a magnetic field prove experimentally that the resultant torque is on the current and not on the magnet.

SLOANE PHYSICS LABORATORY,
YALE UNIVERSITY,
December 3, 1925.

# HALL EFFECT IN EVAPORATED FILMS OF IRON, COBALT· NICKEL, PALLADIUM, AND PLATINUM

By H. B. Peacock

Abstract

**Hall coefficients of evaporated films.**—The Hall coefficients for films of Fe, Co, Ni, Pd, and Pt, 6 to 270$\mu\mu$ thick, deposited on surfaces at a distance less than the mean free path from the evaporating filament, were measured. For films of the ferromagnetic metals, Fe, Co, Ni, the Hall coefficients were respectively 1.4, 4.9, and 7.8 times the values for the corresponding metals in bulk, these factors being in the order of the atomic numbers of the elements. For the paramagnetic, negative Hall effect metals, Pd and Pt, the Hall coefficients were 0.66 and 0.64 times the values for the metals in bulk. The increased Hall effect for the films of the ferromagnetic metals seems closely related to ferromagnetism, although Ni, the least ferromagnetic of the three, shows the greatest variation. Films of Ir and Rh, formed at a distance less than the mean free path from the evaporating filament, showed for Ir a positive Hall effect as in bulk, and for Rh no detectable effect.

**Specific resistance of evaporated films.**—The ratios of the specific resistances of the evaporated films to the specific resistances of the corresponding metals in bulk were found to be 3.1 (Fe), 23.0 (Co), 25.0 (Ni), 4.4 (Pd), 14.0 (Pt), 10.8 (Ir), 12.0 (Rh).

**Saturation of the Hall e.m.f.**—Curves show that the saturation of the Hall e.m.f. occurs for evaporated Fe at a slightly lower magnetic induction than for Fe in bulk; for Co, at about 12,500 gauss as compared with 13,500 for bulk; and for Ni, at about 2500-3000 as compared with 6000 for bulk. It is significant that the magnitudes of these variations from bulk values, like the corresponding variations in the Hall coefficient, occur in the order of these elements in the periodic table.

**Effects of Heating.**—Heating films of the ferromagnetic metals during or after deposition decreased the Hall coefficients and specific resistances to approximately the values for the metals in bulk. It also caused a shift in the value of the magnetic induction necessary to produce saturation of the Hall e.m.f. toward that for the metals in bulk. No effect of heating was observed in the paramagnetic metals.

**Interpretation.**—Evaporated films may differ from bulk metal in the size of the crystalline grains, in the nature of the material between them or in an asymmetry of the crystalline grains which may be an important factor in the causes of the Hall effect.

## Introduction

STEINBERG[1] has shown that for evaporated films of the diamagnetic, negative Hall effect metals, silver and copper, the Hall coefficients were smaller than for the same metals in bulk, while for iron, which shows a positive Hall effect, this coefficient was considerably larger than the

[1] J. C. Steinberg, Phys. Rev. **21**, 22 (1923).

bulk value. Also, for evaporated films of iron, the magnetic induction necessary to produce saturation of the Hall e.m.f. was about half as large as is required for bulk iron. For all of these metals, the Hall coefficients were found to be independent of the thickness. He found the specific resistance of these films to be much larger than for the massive metals, which is in agreement with the work of Weber and Oosterhuis,[2] and also observed the usual critical thickness. He believed the Hall effect to be practically independent of the structure except as it affects the magnetic condition of the crystals. The work of Sorenson[3] shows that the magnetic properties of evaporated ferromagnetic films are different from massive metals but no suggestion is offered as to the cause underlying the results observed by Steinberg on iron.

The present study repeats Steinberg's work on iron and extends the investigation to some other metals of the same group in the periodic table. This group of metals was chosen because it contains both paramagnetic and ferromagnetic elements as well as metals having positive and negative Hall coefficients. It is, therefore, possible to test whether the variations in the Hall effect in evaporated iron are due to its ferromagnetism or to the fact that its Hall effect is positive. If it is due to the latter, cobalt and iridium might be expected to have similar variations; if due to ferromagnetism, nickel, in spite of its negative Hall effect, and cobalt, might be expected to give results similar to iron.

On account of the difficulty of obtaining the metals in the proper form for evaporation, only iron, cobalt, nickel, palladium, platinum, iridium and rhodium were investigated. Of these, the last three and especially the last two were so difficult to evaporate[4,5,6,7] that only a few films were studied. The effect of heat treatment during and after deposition was investigated in some of the films. Films of nickel and cobalt were examined microscopically for crystal structure.

## Experimental

The method of producing the evaporated films was the same as that used by Steinberg[1] with a few improvements in apparatus. Two microscope slides, one above and the other below the filament, were used for the deposition and by means of a heating element beneath the lower

[2] Weber and Oosterhuis, K. Akad. Amsterdam, Proc., **19**, 597 (1917).
[3] A. J. Sorensen, Phys. Rev. **24**, 658 (1924).
[4] J. H. I. Roberts, Phil. Mag. **25**, 271 (1913).
[5] W. Crookes, Roy. Soc. Proc., **A86**, 461 (1911).
[6] Harker and Kaye, Roy. Soc., Proc. **A88**, 522 (1913).
[7] Kaye and Ewen, Roy. Soc., Proc., **A89**, 58 (1913-14).

plate, a temperature difference could be maintained between the two allowing deposition of two films under differing temperature conditions. The approximate temperatures were measured by means of very fine copper-constantin thermocouples held in close contact with the plates.

The pressure in the tube during evaporation was in most cases 0.00075 mm of mercury or less as indicated by a McLeod gauge. In the later experiments, a mercury vapor trap was inserted between the pumps and the tube to eliminate the mercury vapor from the tube. At the pressure used, all parts of the plates were well within the mean free path of the gas in the tube.

The Hall effect, specific resistance and thickness were measured in the same manner and with the same apparatus as was used by Steinberg.[1] In measuring the Hall e.m.f. by the potentiometer method, the average of a number of readings was taken with the field in both directions.

The metals used were the purest that could be obtained in the proper form. Samples of the iron wire of Steinberg[1] and of Sorenson[3] were both used. One piece of nickel wire was obtained from Driver-Harris Company 98.5 percent pure. Another sample was the same as used by Sorensen. In neither the iron nor the nickel was any difference detected in the different samples. The cobalt films were made from cobalt deposited electrolytically on coils of tungsten wire to a depth of 0.25 mm. The differences in melting points and depth of layer probably kept the deposit tungsten-free. The palladium was furnished by Eimer and Amend as "pure." The platinum was Baker and Company's "special commercially pure" and was said to contain 99.5 percent of platinum. Flat strips of iridium and rhodium were produced by grinding down with carborundum wires about 0.7 mm in diameter obtained from the N. E. L. A. Research Laboratory.

## Results

*Hall effect and specific resistance.* The values of the Hall coefficients and specific resistances for the evaporated films are shown in Table I in comparison with the values for bulk metals given by other observers. The Hall coefficient, $R$, is calculated from equation $R=E\epsilon/\beta I$ in which $E$ is the Hall e.m.f., $\epsilon$ the thickness of the film, $I$ the current in the film, and $\beta$ the magnetic induction at right angles to the plane of the film and to the current. The specific resistance is designated by $\rho$ and the subscripts, $e$ and $b$, refer to evaporated and bulk metals, respectively. The values of $R_e$ and $\rho_e$ are the mean values for the number of films indicated deposited on places kept as cool as possible and at temperatures approximately the same. The thickness of the films ranged from 13 to

134$\mu\mu$ for iron, 11 to 166$\mu\mu$ for cobalt, 6 to 270$\mu\mu$ for nickel, 10 to 204$\mu\mu$ for palladium and 6 to 36$\mu\mu$ for platinum. Averaging $R_e$ is justified since it is known to be a constant for all thicknesses for a given metal. In determining $\rho_e$, only those films were included which were thicker than the critical thickness. The values of $R_b$ differ greatly for different observers and there is some question as to what values to use. Smith's[8] values for the first five metals were chosen because his paper also gives other data used.

TABLE I

*The Hall coefficients and specific resistances for evaporated films and for bulk metals.*

| Metal | Evaporated Metals: No. of films | $R_e \times 10^3$ (e.m.u.) Value | Mean dev. | $\rho_e \times 10^6$ (ohms) Value | Mean dev. | Bulk Metals: $R_b \times 10^3$ (e.m.u.) | $\rho_b \times 10^6$ (ohms)[10] | $\frac{R_e}{R_b}$ | $\frac{\rho_e}{\rho_b}$ |
|---|---|---|---|---|---|---|---|---|---|
| 26Fe | 14 | +16 | 3 | 35 | 13 | +11.0[8] | 10 | 1.4 | 3.1 |
| 27Co | 12 | +29 | 5 | 216 | 63 | + 5.88[8] | 9.7 | 4.9 | 23.0 |
| 28Ni | 22 | −99 | 13 | 200 | 58 | −12.7[8] | 7.8 | 7.8 | 25.0 |
| 46Pd | 16 | − 0.57 | 0.07 | 49 | 14 | − 0.86[8] | 11 | 0.66 | 4.4 |
| 78Pt | 8 | − 0.13 | 0.03 | 140 | 50 | − 0.202[8] | 10 | 0.64 | 14.0 |
| 77Ir | 2 | + 0.034 | 0.002 | 66 | 5 | + 0.402[9] | 6.1 | .... | 10.8 |
| 45Rh | 2 | No effect | ..... | 57 | 3 | No data | 4.69 | .... | 12.0 |

*Since iridium and rhodium films were not deposited within the mean free path, these results must not be considered as conclusive.

The Hall coefficients of the evaporated paramagnetic metals are smaller than in bulk as was found by Steinberg[1] for the diamagnetic metals, silver and copper; while for the ferromagnetic metals, these coefficients are larger than in bulk as was found by Steinberg for iron. It must be concluded that the increased Hall effect in evaporated films is a characteristic of ferromagnetism rather than of the sign of the Hall effect. These variations from bulk, being in opposite directions for different metals, practically eliminate the possibility that the observed results are due to an error in assuming the density of the films to be the same as in bulk.

*Effects of heating.* Heating films of iron, cobalt and nickel to a temperature of about 300°C during evaporation had the effect of reducing both $R_e$ and $\rho_e$ nearly to the values for the massive metals. For two films deposited at the same time and under the same conditions except for a temperature difference of about 100°C, both $R_e$ and $\rho_e$ were from 25 to 60 percent smaller for the heated film. The effect of heating after deposition was also investigated in nickel films with approximately the same results.

[8] A. W. Smith, Phys. Rev. **30**, 1 (1910).
[9] H. Zahn, Ann. d. Physik **16**, 148 (1905).
[10] Smithsonian Tables.

No effect of heating was observed in the non-ferromagnetic metals.

*Saturation effects.* It is known that with the ferromagnetic metals the equation $R=E\epsilon/\beta I$ fails to hold for the larger values of $\beta$ and the Hall e.m.f. is said to become saturated. Steinberg[1] has shown that for evaporated iron this saturation occurs at a much lower value of $\beta$ than for bulk iron. This result is confirmed only in a general way. Fig. 1 shows $E\epsilon/I$ plotted as a function of $\beta$ for iron. Curves $A$, $B$, $C$ and $D$, are for evaporated films and the other curve for bulk iron is taken from Smith's[8] paper. These curves show saturations at an induction only slightly less than

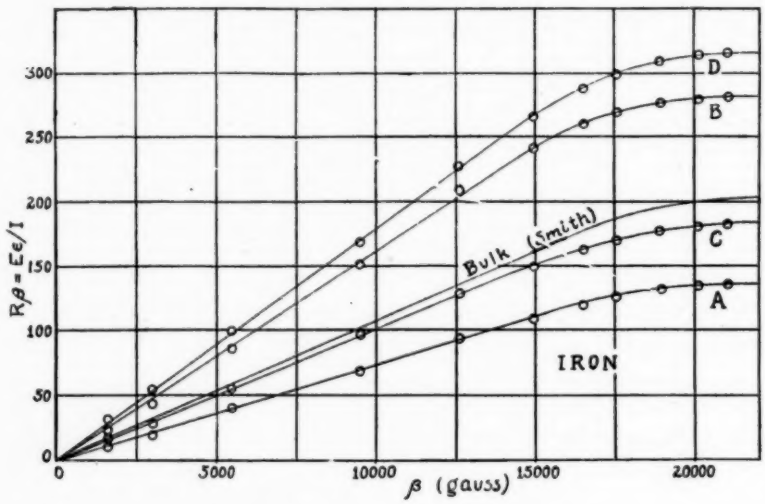


Fig. 1. Saturation of Hall e.m.f. and effect of heat treatment in evaporated iron. $A$ and $C$ are for films heated at a higher temperature.

for bulk metal. Similar curves for cobalt are given in Fig. 2 showing saturation at about 12,500 gauss as compared with 13,500 for bulk. Figs. 3 and 4 show corresponding curves for nickel. For the evaporated films, saturation occurs at about 2500 to 3000 and for bulk at about 6000.

These curves also show effects of heat treatment of the films which are evidenced in two ways. In the first place, the slopes of the curves give the values of the Hall coefficients, which have been shown to be larger for the less heated films. In Fig. 1, curves $A$ and $C$ are for iron films deposited on heated surfaces, $B$ and $D$, on surfaces kept as cool as possible. Films $A$ and $B$ were deposited at the same time. Similar curves for cobalt are given in Fig. 2. Curves $A$ and $B$ are for films made at the same time, $A$ being slightly more heated. $C$ is for a film deposited on an especially hot surface, the definite temperature of which is not known. In the curves of Fig. 3 the pairs $A$,$B$ and $C$,$D$ are for nickel films made at

the same time, respectively, $A$ and $C$ being deposited at the higher temperatures. Fig. 4 shows effects of heating nickel films after deposition.

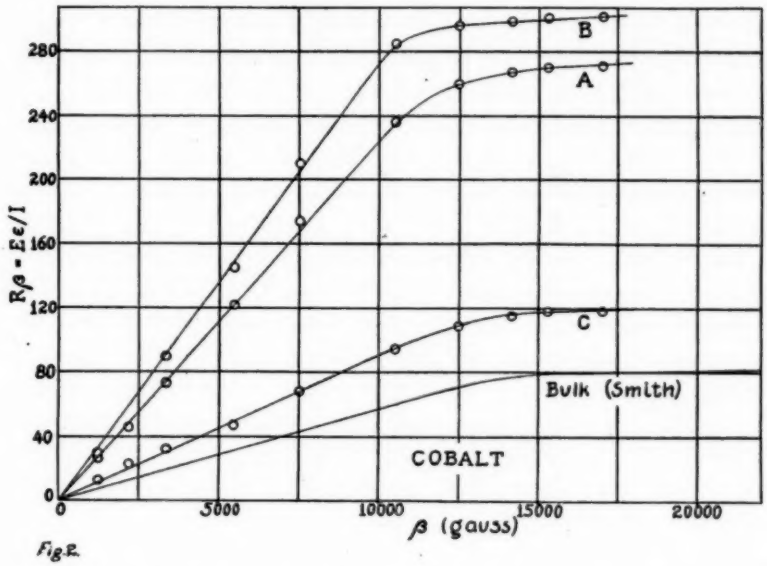


Fig. 2. Saturation of Hall e.m.f. and effect of heat treatment in evaporated cobalt. $C$ is for a film heated at a higher temperature.

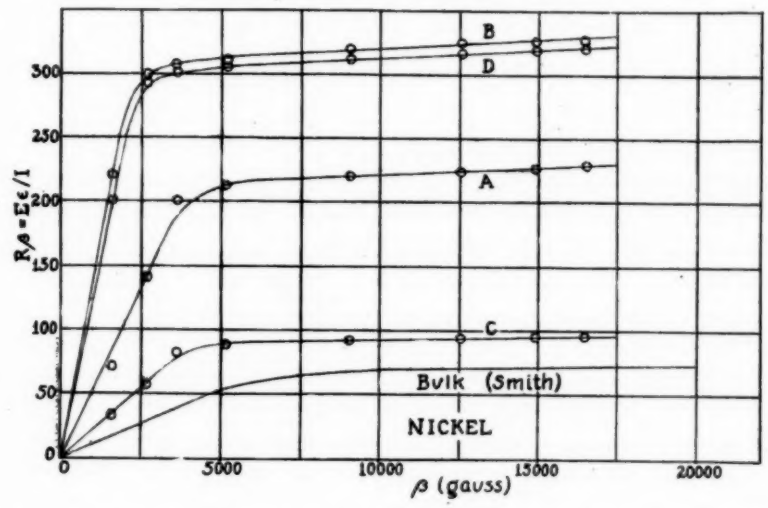


Fig. 3. Saturation of Hall e.m.f. and effect of heat treatment in evaporated nickel. $A$ and $C$ are for films heated at a higher temperature.

Curves $E'$ and $F'$ are for the same films as $E$ and $F$, respectively, taken after heating $F'$ to the softening point of glass and $E'$ to approximately 100°C less.

The second effect of heating shown in these curves is a shift in the magnetic induction necessary to produce saturation. This effect is evidenced only in nickel and cobalt although the effect may be true for iron as well. In the heated films, $C$ for cobalt and $A$, $C$ and $F'$ for nickel, it is evident that the value of the saturating induction is approaching that shown by Smith for bulk metals.

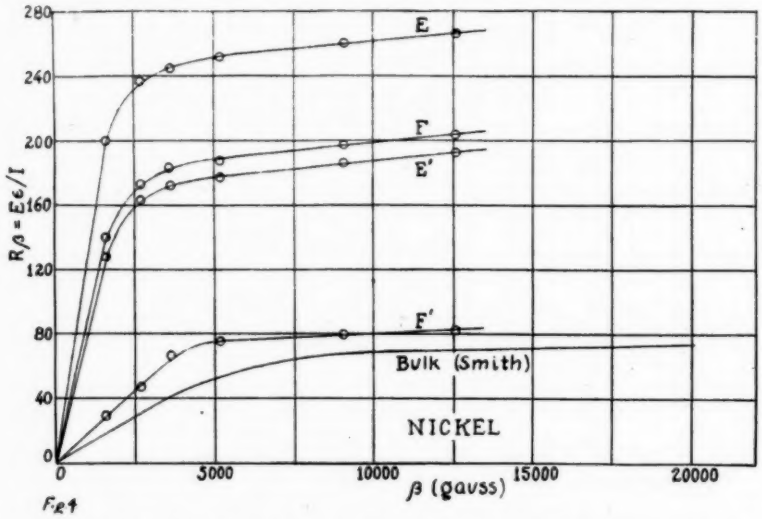


Fig. 4. Saturation of Hall e.m.f. and effect of heat treatment in evaporated nickel. $E'$ and $F'$ are the same films as $E$ and $F$ after heating.

*Microscopic study.* Several of the nickel and cobalt films were examined microscopically for crystal structure. Cobalt films were etched with picric acid and alcohol and nickel films with nitric acid and alcohol. At a magnification of about 300, evidence of structure was observed both by transmitted and by reflected light. Before etching, the films appeared quite smooth with no trace of crystal boundaries.

## Discussion of Results

An interpretation of the above results should be based on a consideration of the probable physical state of evaporated films. Langmuir[11] has suggested the possibility of forming by evaporation either amorphous or crystalline metals depending on the temperature of the surface of condensation. This view is confirmed by the ultramicroscopic investigation of Reinders and Hamburger[12] in which they observed crystal growth in evaporated metal films in changing from liquid air temperature to

[11] I. Langmuir, Am. Chem. Soc., **J38**, 2221 (1916).

[12] Reinders and Hamburger, K. Akad. Amsterdam, Proc. **19**, 958 (1917); *ibid.* **20**, 1135 (1918).

room temperature. They were unable to obtain silver without crystalline structure but tungsten formed a deposit at the temperature of liquid air which could not be resolved. At room temperature, all of the films showed structure. Since the films of the present investigation were all deposited at temperatures considerably above room temperature, they were probably crystalline, a conclusion which is supported by microscopic examination. Three possible ways in which these films may differ from bulk metal are suggested. (1) The observed effects of heating indicate that the crystalline grains may be smaller; (2) since the films were not worked, the material between the crystals may be more porous and spongy, consisting possibly of amorphous metal or of very minute crystals with actual gaps between; (3) because of the method of deposition, the crystalline grains of the films may differ in structure from those of the metal in bulk. The observed results will now be discussed with these ideas in mind.

The high specific resistances found in the present study are in agreement with the results of Weber and Oosterhuis[2] and of Steinberg.[1] Also in the investigation of Reinders and Hamburger[12] referred to above, it was found that increasing crystal size was accompanied by a decrease of resistance which agrees with the results of the heat treatment of iron, cobalt and nickel. Thompson[13] has shown in an investigation on bulk iron that the resistance becomes smaller as the crystals are made larger by annealing. These results seem to lead to the conclusion that the high specific resistance in these evaporated films is due in part to smaller crystalline grains. The heating may also affect a difference in the material between the grains.

An explanation of the mechanism of resistance, which might come under the third possible difference listed above, is suggested by Höjendahl[14] who attempts to explain certain conduction phenomena on the basis of the rotation of the elliptic orbits of the valence electrons. This suggests a need of x-ray and optical investigation of these films.

Greater difficulty is encountered in explaining the Hall effect phenomena because the difference between the Hall coefficients for evaporated films and for metals in bulk are of opposite sign for the ferromagnetic and non-ferromagnetic elements. On the basis of the belief that the crystalline grains are smaller and in less intimate contact in the evaporated films than in bulk, the low Hall coefficients for the non-ferromagnetic metals may be quite well accounted for by assuming that only the crystalline grains contribute to the Hall effect, and that the measured

[13] F. C. Thompson, Phil. Mag. **31,** 357 (1916).
[14] K. Höjendahl, Phil. Mag. **48,** 349 (1924).

current through the film is carried to a larger extent by the gaps between the grains than in bulk, making the current through the crystalline grains themselves smaller than the measured value. Since $R=E\epsilon/\beta I$, using a current which is too large would have the effect of decreasing the value of $R$. With the exception of this error in the value of the current, one should expect the same $R$ for the same material.[15] A further test of the effect of small crystalline grains might be made by thorough heat treatment of these films, if we may assume that high temperature promotes crystal growth.

In the ferromagnetic metals, the situation is evidently quite different, for, as has been shown, the Hall coefficients for evaporated films are considerably larger in absolute value than for the metals in bulk. If the above explanation is valid for the non-ferromagnetic metals, there must be a similar effect in iron, cobalt and nickel, which is hidden by a greater effect in the opposite direction. That this latter effect is closely related to ferromagnetism is evidenced by the fact that nickel, in spite of its negative Hall effect, behaves in a manner similar to iron and cobalt; however, it is difficult to conceive of a change in magnetic properties that would produce the greatest changes in nickel, the least ferromagnetic of the three. That there are differences in the magnetic properties of these films is seen from a consideration of Sorensen's[3] results, but no specific correlation with the present results is suggested.

It seems from the above considerations that there must be some fundamental change in the Hall effect in the separate crystalline grains to account for the observed results on iron, cobalt and nickel. An alteration of the atomic fields of force by some distortion of the crystalline grains or other asymmetry resulting from the conditions of deposition may offer a possible explanation.

The present results on the saturation of the Hall e.m.f. might seem to indicate a lack of proportionality between the Hall effect and the intensity of magnetization in iron, cobalt and nickel as found for these metals in bulk by DuBois[16] and Kundt.[17] This is not a necessary conclusion, however, since no magnetic measurements have been made on thin films magnetized normally to the plane as in Hall effect measurements. Inasmuch as $H$ must be small, on account of the demagnetizing effect in a thin film, $\beta$ is probably nearly equal to $4\pi I$, but the actual relation is unknown.

[15] G. W. Stewart, Phys. Rev. **17**, 232 (1921).
[16] H. G. J. DuBois, Ann. d. Physik **31**, 491 (1887).
[17] A. Kundt, Ann. d. Physik **49**, 257 (1893).

Smith's[8] results on the variation of the Hall effect with temperature should be noted. The Hall coefficients with increasing temperature increased for the ferromagnetic metals and decreased for the non-ferromagnetic metals. This is similar to the above results. Also, he observed a similar decrease in the value of the magnetic induction necessary to produce saturation of the Hall e.m.f. in the ferromagnetic metals.

The average values of the Hall coefficients for evaporated iron, cobalt and nickel are 1.4, 4.9 and 7.8 times, respectively, the values for the metals in bulk. The magnetic induction necessary to produce saturation of the Hall e.m.f. is for nickel about 60 percent lower than in bulk, for cobalt about 8 percent lower and for iron only slightly lower. These results seem significant when we consider that in both cases these variations are in the order of the atomic number of these elements and may indicate some fundamental relation to atomic structure. If the increase in the Hall effect is due to an asymmetry in the crystals, then there is a causal atomic change related to the increase in atomic number.

In conclusion, the writer wishes to express his appreciation to the members of the Department of Physics of the State University of Iowa for their assistance and interest, and especially to Professor G. W. Stewart under whose direction the work was carried on.

PHYSICAL LABORATORY,
UNIVERSITY OF IOWA.
December 23, 1925.

# NOTE ON THE MAGNETIC PROPERTIES OF RARE EARTH OXIDES

By E. H. Williams

Abstract

The *magnetic susceptibility* of three very pure rare earth oxides, namely those of cerium, holmium and praseodymium, have been measured and found to be $0.14\times10^{-6}$, $229\times10^{-6}$, and $13.4\times10^{-6}$ dyne cm per gm respectively. The spectroscopic and atomic weight analyses for the purity of one of the elements have been checked by magnetic susceptibility measurements. The latter were found to be in good agreement with the former.

*Resolution numbers* obtained from a theory based by Jakob Kunz on the anomolous Zeeman effect have been compared with magnetic susceptibilities obtained by the author in this and previous papers and found to be consistent therewith.

THE scarcity of rare earths of very high degree of purity renders it desirable to make magnetic measurements on them when once they are obtained. For several years B. S. Hopkins of the Chemistry Department of this university, together with his assistants and students, has been engaged in the purification of rare earth elements and in chemical measurements upon their compounds. In previous articles[1] the author described the results of magnetic susceptibility measurements on some rare earth oxides at both high and low temperatures. Since the time of the above measurements the oxides of three more elements, namely, cerium, holmium and praseodymium, have been obtained in a very high state of purity.

The Curie balance method employed was the same as that used in the previous measurements. No attempt was made to vary the temperature, all measurements being made at room temperature and reduced to 20°C by applying Curie's law for paramagnetic substances. In no case did the room temperature differ by more than two or three degrees from 20°C.

TABLE I

*Magnetic susceptibilities*

Temperature, 20°C.

| Substance | $\chi\times10^{6}$ dyne cm/gm |
|---|---|
| Cerium oxide | 0.14 |
| Holmium oxide | 229.0 |
| Praseodymium oxide | 13.4 |

[1] E. H. Williams, Phys. Rev. **12**, 158 (1918); **14**, 348 (1919).

The results for the magnetic susceptibility per gram for the three oxides are shown in Table I. The purity of each of the oxides was 99.5 percent or better.

In the first paper referred to above the author pointed out that when only two substances are involved the magnetic test is reliable in the analysis of substances. About the time the above measurements were being made, Dr. John Wierda of the Department of Chemistry was determining the atomic weight of praseodymium. To check the spectroscopic and atomic weight analyses which he was making, magnetic susceptibility tests were made. Samples of praseodymium oxide at successive stages in the process of purification were measured. The results are shown in Table II.

TABLE II

*Magnetic susceptibility of samples of praseodymium oxide*

| Sample | $\chi \times 10^6$ dyne cm/gm |
|---|---|
| No. 23 | 9.3 |
| No. 24 | 13.26 |
| No. 25 | 13.23 |

These magnetic values indicate that sample No. 24 was as pure as sample No. 25. Atomic weight and spectroscopic tests gave the same results.

In a theory soon to be published Jakob Kunz arrives at certain numbers which give the resolution in the Stern and Gerlach experiment, i.e., the number of atomic beams which one would expect on the basis of the anomalous Zeeman effect. This resolution number is intimately connected with the magnetic susceptibility. In order to test this theory, the numbers that apply to the rare earth elements measured in this and the two previous papers are compared with the corresponding values of susceptibility in Table III.

TABLE III

| Substance | Resolution No. | $\chi \times 10^6$ dyne cm/gm (20°C) |
|---|---|---|
| Lanthanum oxide | 4 | −0.4 |
| Yttrium oxide | 4 | 0.53 |
| Cerium oxide | 9 | 0.14 |
| Samarium oxide | 9 | 6.0 |
| Praseodymium oxide | 12 | 13.4 |
| Neodymium oxide | 13 | 29.6 |
| Gadolinium oxide | 13 | 130.1 |
| Erbium oxide | 21 | 189.0 |
| Dysprosium oxide | 21 | 234.0 |
| Holmium oxide | 22 | 229.0 |

In general it is seen that there is good proportionality between the resolution number obtained by Kunz and the magnetic susceptibility values found by the author. The discrepancies may be due to the fact that the oxides were used instead of the elements.

The fact that, in the theory of Kunz, the resolution numbers of the atom for some of the rare earth elements are larger than those of any other elements gives special significance to their magnetic properties. The resolution numbers of erbium, dysprosium and holmium are more than twice that of iron and therefore we should expect these elements in the pure state to have very high magnetic susceptibilities. Thus far it has been impossible to obtain any of these elements in the pure state and thus to test this part of the theory.

PHYSICS LABORATORY,
UNIVERSITY OF ILLINOIS.
December 23, 1925.

# ACOUSTIC TRANSMISSION WITH A HELMHOLTZ RESONATOR OR AN ORIFICE AS A BRANCH LINE

By G. W. Stewart

Abstract

The author's theory of acoustic transmission in a conduit with a branch line is extended to the cases of a Helmholtz resonator and an orifice attached as branches. The effect of viscosity is included in an approximate form.

*Helmholtz resonator.* The components of the point impedance of the branch are found to be: $Z_1=(2\omega\rho\mu)^{\frac{1}{2}}(\pi R^2)^{-1}L$ and $Z_2=\rho\omega/c-\rho a^2/V\omega+(2\omega\rho\mu)^{\frac{1}{2}}L(\pi R^2)^{-1}$, wherein $\omega=2\pi\times$frequency, $\rho$ is density, $\mu$, viscosity, $R$, radius of orifice, $L$, its equivalent length, $a$, the velocity of sound, and $V$, the volume of the resonator. *Experimental verification of the theory is obtained* and in addition there is found the approximate value of a simple channel equivalent in viscosity effect to the orifice of the resonator. *The serious diminution of transmission* caused by the resonator is not limited to the neighborhood of the resonance frequency but *extends over a range of two octaves.*

*Orifice.* The components of the point impedance of the branch are found to be the same as those of the Helmholtz resonator, but with the term containing $V$ omitted. Experimental verification is obtained with sizes of orifices from 7 percent to 42 percent the diameter of the conduit. *The radiation from such orifices is shown to be relatively small* and thus their impedances are practically imaginary. *Transmission* in the conduit *increases* with *frequency.* Action of *orifices* in *musical instruments* is explained by the theory. Relative values of radiation and transmission are shown by computation.

THIS is a continuation of the theory of acoustic transmission with branch lines and its purpose is a detailed investigation of the cases cited in the title.

*Helmholtz resonator.* In the previous article[1] was deduced the general expression for transmission,

$$(P'/P)^2=[(Z_1^2+Z_1\rho a/2S+Z_2^2)^2+(\rho aZ_2/2S)^2]\times[(Z_1+\rho a/2S)^2+Z_2^2]^{-2} \tag{1}$$

wherein $P'$ and $P$ refer to the actual pressure at the junction point of branch and conduit, $Z_1$ and $Z_2$ the components of the point impedance of the branch line, $Z=Z_1+iZ_2$, $\rho$ is the density of the medium, $a$ is the velocity of sound and $S$ is the area of the conduit.

It was also shown that, neglecting viscosity, for the case of the Helmholtz resonator, $Z_1=0$ and $Z_2=\rho\omega/c-\rho a^2/V\omega$ in which $\omega$ is $2\pi$ times the frequency, $c$ is the conductivity of the orifice and $V$ is the volume of the chamber. The above general equation is then reduced to

$$|P'/P|^2=\{1+[4S^2(k/c-1/kV)^2]^{-1}\}^{-1} \tag{2}$$

[1] Stewart, Phys. Rev. **26**, 688 (1925).

The approximation will be made that the effect of the viscosity in the orifice is equivalent to the viscosity in a channel of length $L$. The equation of motion of the gas in such a channel in which the layer adhering to the wall is small in comparison with the diameter, is[2]

$$\rho\ddot{X}dx+(\tfrac{1}{2}\omega\rho\mu)^{\frac{1}{2}}\quad O/S(\dot{X}+1/\omega\cdot\ddot{X})dx=\rho a^2\partial^2X/\partial x^2dx \tag{3}$$

Herein $O$ is the perimeter, $S$ is the area, $\mu$ is the coefficient of viscosity, and $dx$ is the element of length of the tube. On the basis of assumption already stated the velocity at a distance from the walls may be regarded as $\dot{X}/S$. For a channel as short as here supposed, $\dot{X}$ may be regarded as the same throughout the length $L$ at any instant, and consequently $\ddot{X}$ and $\partial^2X/\partial x^2$, which are proportional to $X$ in a tube of constant cross-section, may be also regarded as similarly constant. After integrating over $L$, dividing by $S$, and substituting the values of $O$ and $S$ in terms of the radius $R$, the following is obtained

$$\begin{aligned}[\rho L/S+(2\omega\rho\mu)^{1/2}(\pi R\omega)^{-1}L]\,\ddot{X}+(2\omega\rho\mu)^{1/2}(\pi R^3)^{-1}L\dot{X}\\ =\rho a^2\partial^2X/\partial x^2\cdot L/S\ .\end{aligned} \tag{4}$$

From the fundamental equations of plane waves, $\rho a^2\partial X/\partial x=-S\partial p/\partial x$, $p$ being pressure, and consequently the right hand member is the difference in pressure acting on the mass in the channel. This is $P'-\rho a^2X/V$, if $X$ is the inward displacement into a Helmholtz resonator of volume $V$ for the pressure exerted on the capacitance is $\rho a^2X/V$. Since $\rho L/S$ is really the inertance of the channel, it should be replaced by $\rho/c$ of the actual orifice. When these substitutions are made and the relationship $\ddot{X}=i\omega\dot{X}=-\omega^2X$ is applied, there results,

$$\begin{aligned}\{(2\omega\rho\mu)^{1/2}(\pi R^3)^{-1}\cdot L+i[\rho\omega/c-\rho a^2/V\omega+(2\omega\rho\mu)^{1/2}\\ \times(\pi R^3)^{-1}\cdot L]\}\,\dot{X}=P'\end{aligned} \tag{5}$$

The coefficient of $\dot{X}$ is the impedance of the resonator and consequently we may write at once the values,

$$\begin{aligned}Z_1&=\ ^{1}\!/_{2}((2\omega\rho\mu)^{1}\pi R^3)^{-1}\cdot L\ ,\\ Z_2&=\rho\omega/c-\rho a^2/V\omega+(2\omega\rho\mu)^{1/2}\cdot L(\pi R^3)^{-1}\ .\end{aligned} \tag{6}$$

If the values of Eq. (6) be now substituted in Eq. (1), the transmission can be computed for any frequency. $Z_1$, upon which dissipation obviously depends, vanishes with $\mu$ as would be anticipated. $Z_2$ depends upon the inertance and capacitance of the resonator, and the effect of viscosity is virtually an addition to the inertance or the subtraction from the capacitance of a function varying with $\omega$ and $\mu$.

[2] Rayleigh, Theory of Sound, Vol. II, §347, Eq. (10).

*The orifice.* If there is an orifice without the resonator, the correct expression for the pressure acting on the mass in the channel is no longer $(P'-\rho a^2X/V)$ but $(P'-\rho(k^2a/2\pi)\dot{X})$ where the second term in the latter[3] is the dissipative force at the opening. Making this substitution in (4) and proceeding as before, there is obtained

$$\{\rho k^2a/2\pi+(2\omega\rho\mu)^{1/2}(\pi R^3)^{-1}\cdot L+i[\rho\omega/c+(2\omega\rho\mu)^{1/2}\times(\pi R^3)^{-1}\cdot L]\}\dot{X}=P' . \tag{7}$$

Thus,

$$\begin{aligned} Z_1&=\rho k^2a/2\pi+(2\omega\rho\mu)^{1/2}(\pi R^3)^{-1}\cdot L ,\\ Z_2&=\rho\omega/c+(2\omega\rho\mu)^{1/2}(\pi R^3)^{-1}\cdot L . \end{aligned} \tag{8}$$

Thus, dissipation depends upon both viscosity and radiation, while the inertance is affected by the presence of viscosity. The values of Eq. (8) may now be substituted in Eq. (1) and the transmission computed for any frequency.

*Experimental and theoretical results.* The circles on the plot of Fig. 1 show the experimental values of $|P'/P|^2$ as taken by an ear comparison method of intensity measurement[4] which is sufficiently accurate for the present purpose. In computing the theoretical results, the only uncertainty is the selection of the value $c$, for the conductivity of the orifice. Were it an orifice in an infinite plane wall, we have given the well-known theoretical[5] inferior and superior limits for the corrections to the length of one end of the channel; namely $.785R$ and $.85R$. But it is known that the correction without a flange is distinctly less, approximately $.6R$. The correct value for the present cases of an opening in a tube with or without a chamber, is not known and therefore must be selected. The writer has somewhat arbitrarily chosen the inferior theoretical flange value, $.785R$, making the total correction of length $2\times.785R$ or $\pi R/2$. The error will not be great and can be ascertained by later experimental refinements. The value of $c$ is thus $\pi R^2/(L+\pi R/2)$.

The curve drawn in Fig. 1 has been computed in accord with Eq. (1) with the values of $Z_1$ and $Z_2$ given by Eqs. (6). It is found by trial that for any value of $L$ comparable to the length of the orifice, the curve is not altered by assuming $\mu=0$ or by using (2). The dotted curve with ordinates designated on the right shows the computed response of the same Helmholtz resonator in the open with a source of sound producing

[3] Rayleigh, Theory of Sound, Vol. II, §311.

[4] The method used is really an improvement of that discussed in the Phys. Rev. **20**, 528 (1922), and will be described in a later article.

[5] Rayleigh, Theory of Sound, Vol. II, 183.

a pressure $P$ at the resonator when the orifice is closed. If the internal variation of pressure is $G$, the ordinates represent[6] $|G/P|^2$ and indicate the sharpness of response of such a resonator as ordinarily used.

Fig. 2 is similarly prepared for the case of a small orifice, the chamber being reduced in size also so that the computed minimum transmission without viscosity is at approximately the same frequency as in Fig. 1. Curve $a$ is the theoretical curve omitting viscosity and curve $b$ the computed one using the values given in Eq. (6), assuming $\mu = .00018$ and $L = .09$ cm or six times the actual length of the orifice.

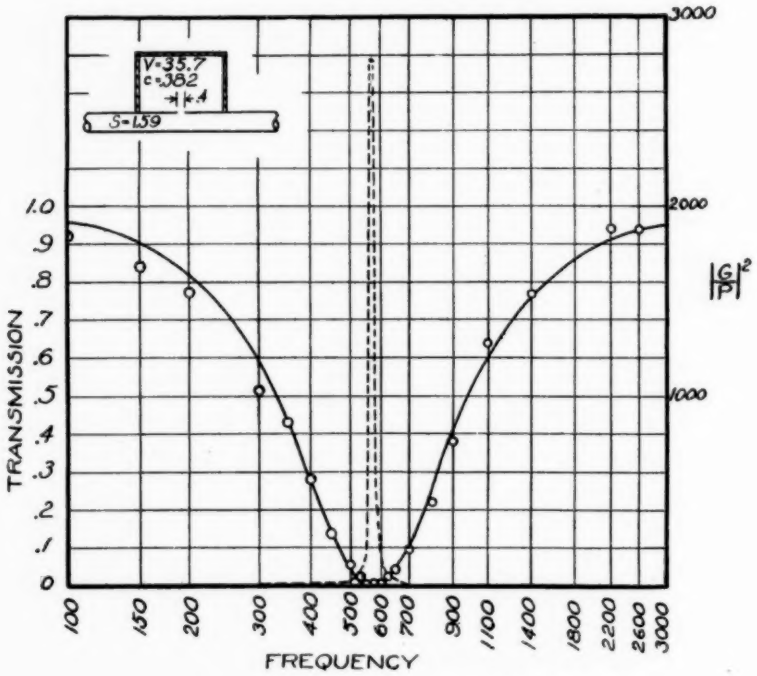


Fig. 1. Helmholtz resonator as a branch.

There are several points of interest in the comparisons. First, it is noticed that the resonator seriously influences transmission for more than an octave in either direction. This is hardly to be anticipated from the sharpness of the action of the resonator as ordinarily used and as indicated by the dotted curve in Fig. 1. Second, the comparison of experiment with theory justifies the assumption of an equivalent channel as a close approximation to the effect of viscosity in the region of the frequency of minimum transmission. This experiment is, in fact, an

[6] Rayleigh, Theory of Sound, Vol. II, §311.

$$|G/P|^2 = [(1-k^3V/c)^2 + (k^3V/2\pi)^2]^{-1}$$

approximate method of determining the equivalent length of this channel in this frequency region. Computation shows that a change of 10% in the assumed $L$, will produce a noticeable inferiority of agreement with experiment. Third, the comparison of theory and experiment gives entire confidence in the former. The variations from theory are probably caused by experimental inaccuracies.

Experiments with an orifice are perhaps more interesting because such cases occur frequently. Fig. 3 shows the dimensions of the orifices used, the square root of the observed transmission is indicated by marked points and the computations from theory are represented by a full line curve. The values last named are obtained by substituting the values

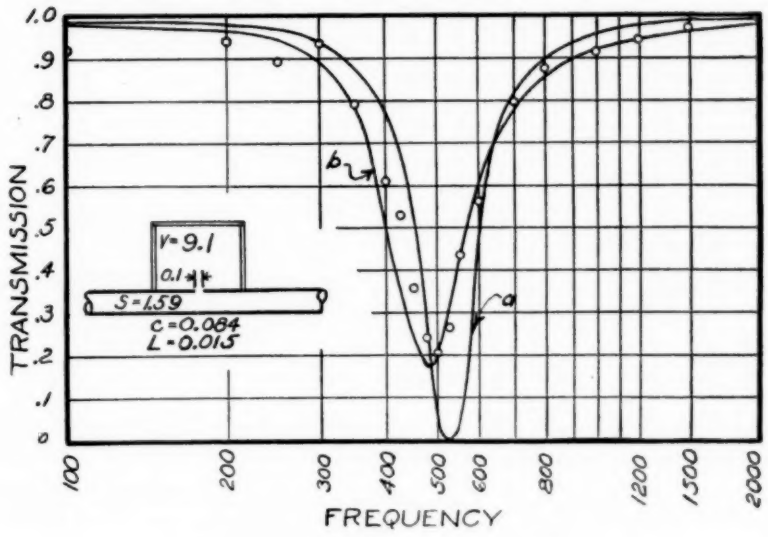


Fig. 2. Effect of viscosity in the orifice.

of $Z_1$ and $Z_2$ of Eq. (8) in Eq. (1) but first assuming $\mu = 0$. If the value, $\mu = .00018$, be used, no appreciable change in the curves results, even with the smallest orifice. Curve 4, Fig. 3, does not agree with the observed points and this is doubtless because, as the orifice becomes larger, the value of $c$ is less and less approximately the value for the orifice opening into an infinite plane. If its value is increased arbitrarily to 0.74, the agreement with observations is satisfactory. Obviously there is here suggested a method of measuring $c$. It should be mentioned that this experimental value of 0.74 proves to give agreement of theory and experiment in the case of a Helmholtz resonator as a branch having the same orifice. In order to test the influence of radiation from the orifice into the open, the value of $Z_1$, Eq. (8), is placed equal to zero in Eq. (1).

The resulting changes in the lowest curve are shown by the dotted curve in Fig. 4. This is the case of the largest orifice. *The change is surprisingly small.* It should be stated that in the theory the radiation is assumed to occur from a hole in an infinite plane wall, an approximation not unsatisfactory because of the small amount of radiation. Fig. 4 also shows the data and computations at higher frequencies using the largest orifice.

The above comparisons of theory and experiment present several points of interest. First, orifices in a conduit diminish transmission with a magnitude increasing with decreasing frequency. Second, neither

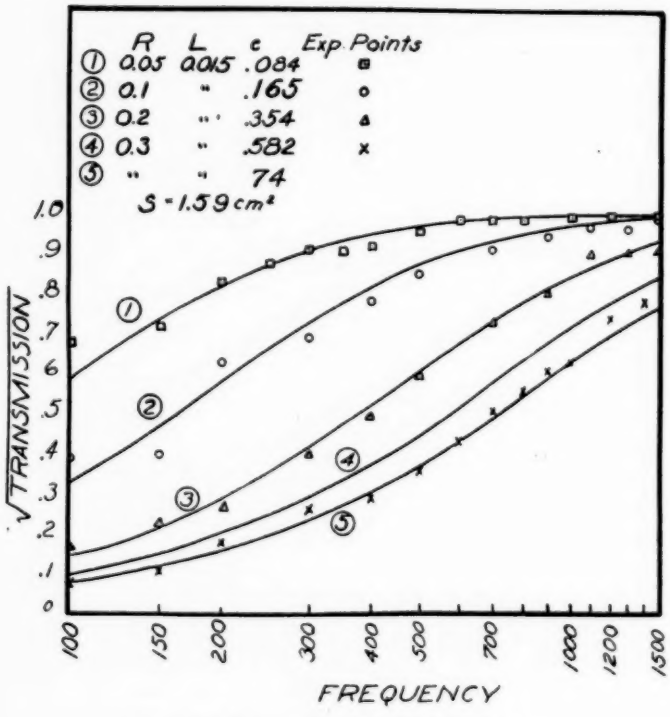


Fig. 3. Orifices as branches of a conduit.

viscosity nor radiation seriously influences the value of transmission in the cases cited. Third, the diminution in transmission is not caused by a loss of energy but by the inertance of the orifice which produces a reflected wave.

While the foregoing refers to a wave transmitted in one direction, the general effect produced in a resonant musical instrument can be understood. Consider the opening of a key on a clarinet. There is a reflected wave from the opening, as if it were an open end of a pipe and this condition, as is well known, enables the *resonant* tone for that pipe length to

be established. The intensity of the incident wave becomes very large and consequently the sound escaping from the bell of the clarinet is made sufficiently large. Thus it is noticed that most of the sound escapes from the bell of the clarinet and not from the orifice. This wave reflected from the orifice would be a detriment to the establishment of any tone not in resonance. The radiation from the hole is small in comparison with the transmission along the tube and subsequent radiation from the bell. This phenomenon may be explained in the following terms: the

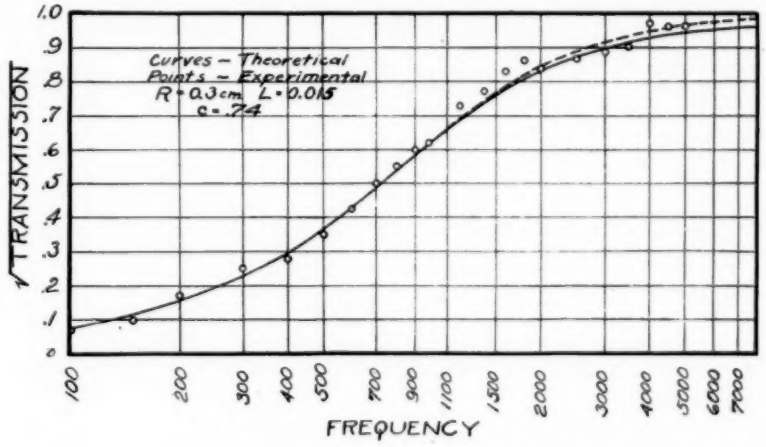


Fig. 4. Difference in transmission caused by radiation.

impedance of an orifice is approximately, but not entirely, imaginary, and hence practically wattless, whereas the impedance of an infinite pipe or conduit is real and has no wattless component. A number of years ago Boehm[7] noted that the transmission along the axis of the flute was much greater than through an orifice having an area as large as the cross-section of the conduit. The above discussion supplies the explanation.

I wish to acknowledge the valuable assistance of Mr. W. D. Crozier in these experiments.

DEPARTMENT OF PHYSICS,
UNIVERSITY OF IOWA,
January 5, 1926.

[7] Theobald Boehm, "On the Flute," translated and published by D. C. Miller. See p. 25.

# THE TUBE AS A BRANCH OF AN ACOUSTIC CONDUIT: THE SPECIAL CASE OF THE QUINCKE TUBE

By G. W. Stewart

Abstract

The author's theory of acoustic transmission in a conduit with a branch line is extended by obtaining the thoretical values of the components of the point impedance of a closed and of an open tube.

*The branch an open tube.* The components of the impedance of the branch are $Z_1=(\rho\omega k/2\pi)D^{-1}$ and $Z_2=\rho\omega/c_0+[\rho\omega/k\sigma\cdot\sin kl\cos kl+\rho\omega/c\,(\cos^2 kl-\sin^2 kl)-k\sigma\rho\omega(k^2/4\pi^2+1/c^2)\sin kl\cos kl]D^{-1}$, wherein $D=k^2\sigma^2(k^2/4\pi^2+1/c^2)\sin^2 kl+\cos^2 kl-2k\sigma/c\sin kl\cos kl$, $\rho$ is the density, $\omega=2\pi\times$ frequency $=k\times$(sound velocity), $c_0$ and $c$ are the conductivities of the conduit orifice and distant orifice respectively, $\sigma$ is the area and $l$ is the length of the tube. These values of impedances are used in computing the transmission through the conduit in accordance with fundamental theory (Phys. Rev. **5**, 26, 688 (1925)). A comparison with experiment shows agreement. The transmission when the tube is at resonance is very different from the case of the same orifice without the tube.

*The branch a closed tube.* The above values of impedance reduce to $Z_1=0$, and $Z_2=\rho\omega/c_0-\rho a/(\sigma\tan kl)$. The theory of transmission thus established is that of the Quincke tube. The selectivity is not sharp as is the cylinder in the open, the effect being marked over a total range of an octave. The effect of viscosity in increasing the minima of transmission is shown.

*The branch an impedance of any kind.* If the point impedance of the attachment be known, then to its imaginary component may be added $\rho\omega/c_0$ and both components used in the original formula for transmission. Also if such an attachment is by means of a tube of constant cross section, the theory will give the transmission.

## Theory

THE theory previously established for the influence of branch lines on acoustic transmission[1] shows that the ratio of the energy flow in the transmitted wave to that in the incident wave is

$$[(Z_1^2+Z_1\rho a/2S+Z_2^2)^2+(\rho aZ_2/2S)^2][(Z_1+\rho a/2S)^2+Z_2^2]^{-2}.$$

Here $\rho$ is the density of the medium, $a$ the sound velocity, $S$ the area of the conduit and $Z_1$ and $Z_2$ are the components of the point acoustic impedance $Z=Z_1+iZ_2$, defined as the ratio of the excess pressure and the volume-current, all three quantities being regarded as complex. The computed values depend upon a knowledge of $Z_1$ and $Z_2$. The purpose of this contribution is to establish the expression for the values of $Z_1$ and $Z_2$ for any length of tubular side branch, open or closed, and in fact

[1] Stewart, Phys. Rev. **26**, 688 (1925).

for any attachment, the point impedance of which is known, connected to the conduit by a tube of constant cross-section.

Consider a tube of diameter small in comparison with a wave-length, making the wave front perpendicular to the axis. Let the displacement be $\xi$, positive with the axis $x$ of the tube. It can then be shown[2] that, in case of a simple harmonic vibration:

$$\partial^2 p/\partial x^2+\partial/\partial x\,(\log\sigma)\cdot\partial p/\partial x+k^2p=0 \tag{1}$$

$$\partial^2\xi/\partial x^2+\partial/\partial x\,(\log\sigma)\cdot\partial\xi/\partial x+\partial^2/\partial x^2\,(\log\sigma)\cdot\xi+k^2\xi=0 \tag{2}$$

wherein $p$ is the pressure, $k$ is $2\pi$ divided by the wave-length, and $\sigma$ is the area of the tube.

The solutions of the equations take the form[2]

$$p_2=a'p_1+b'X_1\,,\qquad X_2=c'p_1+d'X_1 \tag{3}$$

wherein the subscripts, 1 and 2, refer to the open ends of the tube at $x_1$ and $x_2$, $x_2>x_1$, $X$ is $\sigma\xi$, or the volume displacement, and $a'$, $b'$, $c'$ and $d'$ are constants the general values of which are indicated by Webster.[2] Eq. (3) may be written in terms of $X_1$ and then from the two may be obtained the following:

$$Z_1'=p_1/\dot{X}_1=(i\omega Z_2'd'-b')(c'\omega^2Z_2'+i\omega a')^{-1}\,. \tag{4}$$

Herein $Z_1'$ and $Z_2'$ are complex impedances defined by $Z_1'=p_1/\dot{X}_1$ and $Z_2'=p_2/\dot{X}_2$. Assuming a cylindrical tube, $\sigma$ is constant, and Webster's values of the constants become: $a'=d'=\cos kl$, $b'=(\beta/\sigma)\sin kl$, $c'=-(\sigma/\beta)\sin kl$, if $\beta$ is written for $\rho a^2k$. Eq. (4) then becomes

$$\begin{aligned}Z_1'=(\beta/\sigma)\,&[i\omega Z_2'\cos kl-(\beta/\sigma)\sin kl]\\ &\times[i\omega(\beta/\sigma)\cos kl-\omega^2Z_2'\sin kl]\end{aligned} \tag{5}$$

If the attachment of the tube to the conduit be at $x_1$, then $Z_2'$ is the point impedance of the other end, $x_2$. Eq. (5) then gives the point impedance of the tube for any attachment at its terminus, if the point impedance of the attachment, $Z_2'$, is known. In the actual arrangement here discussed, there should be added to $Z_1'$, the inertance of the orifice into the conduit. As shown in an earlier paper,[3] this is $\rho\omega/c_0$, the subscript of $c_0$ being used to distinguish it from the conductivity of the open end.

If the distant end of the tube is open, the value of $Z'_2$ is that of an orifice,[3] $(\rho k^2a/2\pi+i\rho\omega/c)$, wherein $a$ is the velocity of sound and $c$ is the conductivity of the orifice.

[2] For example see A. G. Webster, Proc. Natl. Acad. Sci. **5**, 275–282 (1919), Eqs. (21), and the solutions Eq. (24).

[3] Stewart—preceding paper in this issue. In Eq. (8), if $\mu=0$, the value quoted in second reference follows by adding $Z_1$ and $iZ_2$.

When to Eq. (5) there is added $\rho\omega/c_0$ and there is inserted the value of $Z_2'$ for the open end, the following components of impedance are readily found:

$$Z_1=(\rho\omega k/2\pi)D^{-1}$$

$$Z_2=\rho\omega/c_0+\left[\rho\omega/k\sigma\cdot\sin kl\cos kl+\rho\omega/c(\cos^2 kl-\sin^2 kl)-k\sigma\rho\omega\left(\frac{k^2}{4\pi^2}+\frac{1}{c^2}\right)\cdot\sin kl\cos kl\right]\times D^{-1} \tag{6}$$

if $$D=k^2\sigma^2\left(\frac{k^2}{4\pi^2}+\frac{1}{c^2}\right)\sin^2 kl+\cos^2 kl-\frac{2k\sigma}{c}\sin kl\cos kl\ .$$

If the cylinder is closed, $Z_2'=\infty$, $Z_1'=-(i\rho a/\sigma)\cot kl$, and

$$Z_1=0\ ,\quad \text{and } Z_2=\rho\omega/c_0-\rho a/(\sigma\tan kl) \tag{7}$$

It is to be observed that in obtaining Eqs. (6) the value of $Z_2'$ was obtained as if the cylindrical tube opened into an infinite plane. Any attachment whatever can be made at this point and, if its point impedance is known equations corresponding to (6) can be obtained by appropriate substitution of $Z_2'$ in Eq. (5).

Given the values of $Z_1$ and $Z_2$ in any case, the substitution may be made in the value of the transmission given in the first paragraph. The selection of the values of $c_0$ for the entering orifice of the conduit and of $c$ for the other end is a matter of judgment as experimental knowledge is lacking. In a previous article[3] it was explained that the correction to the length at the conduit end would be made $.785R$ or $c_0=4R$. Inasmuch as experimental values of the correction[4] for the open end average about $.57R$, the conductivity of the open end would be $c=\pi R^2/.57R=5.5R$. These two values will be used in the computations.

## Experimental Results

As illustrative of the application of the equations, the experimental results for the closed and the open tube will be shown.

Fig. 1 shows the square root of the transmission in the case of the closed tube. The points represent observations made by an ear comparison method previously described,[4] and the full line represents the computations using the values of Eq. (7) in the expression

$$[(Z_1^2+Z_1\rho a/2S+Z_2^2)+(\rho a Z_2/2S)^2]^{1/2}[(Z_1+\rho a/2S)^2+Z_2^2]^{-1}$$

or the square root of the transmission. The difference is two-fold. First, the experimental curve does not have a minimum of zero. This is prob-

[4] See Barton, Text Book of Sound, p. 251.

ably caused by dissipation through viscosity. Second, the computed points are displaced toward smaller frequencies. This is accounted for by our consistent adherence to the value of $c_0 = 4R$. It is obvious that,

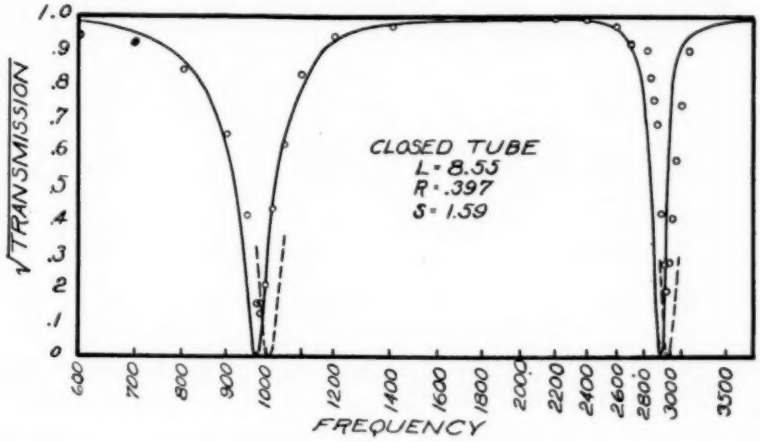


Fig. 1. The closed tube as a branch.

although this value may be satisfactory where the orifice into the branch is small in comparison to the area of the conduit, yet in the present case it cannot be a correct value, for the end does not open out into free space.

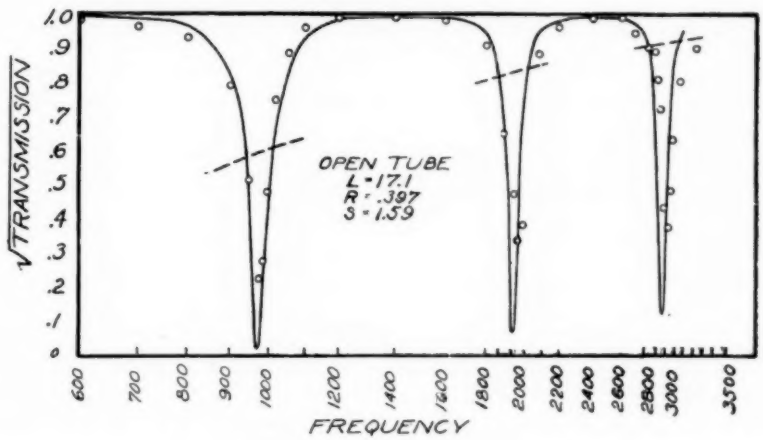


Fig. 2. The open tube as a branch.

In this value of $c_0$ must be acknowledged an unavoidable approximation. The dotted minima indicate the computations with $c_0 = \infty$. Undoubtedly viscosity accounts for the increase with frequency of the transmission at

the minimum. The variation between experiment and theory is sufficiently small to indicate the essential accuracy of the theory.

Fig. 2 shows the case of an open tube approximately twice as long as the closed tube in Fig. 1. The effect of viscosity should be greater and in addition there is radiation. The curves seem to verify this. The dotted line shows what would be expected if $l$ were zero. It is obvious that it does not and should not give the same transmission as that at the minimum with the resonating tube. The minima increase with frequency, indicating an increased radiation and increased viscosity. Comments concerning the value of $c_0$ would be similar to those just given in regard to Fig. 1.

## Quincke's Tube

It is evident that this theory of the closed cylinder covers the case of a Quincke tube which is merely a side tube that decreases transmission at its resonance frequency. Without viscosity the reduction is complete irrespective of the diameter of the branch, but in practice it is necessary to use several branches[5] in order to reduce this resonance frequency to inaudibility. The results of both theory and experiment in Fig. 1 indicate the effectiveness of such a Quincke tube and its range of selectivity. When using more than one such tube in the branch, it is necessary to place them one-half wave-length apart, for then the destructive waves from them agree in phase. It is not necessary, as Kreuger[6] states, to place them at any fixed distance from the source provided there is sufficient damping to prevent the backward waves from influencing the source.

I wish to acknowledge the valuable assistance of Mr. W. D. Crozier in these experiments.

Department of Physics,
University of Iowa,
January 5, 1926.

[5] The method used is really an improvement of that discussed in the Phys. Rev. **20**, 528 (1922), and will be described in a later article.

[6] Kreuger, Philos. Stud. **17**, p. 223, (1901).

# THE DURATION OF THE TRANSIENT STATE IN THE SETTLING OF SMALL PARTICLES

By Warren Weaver

Abstract

The general theory of the settling of small particles in a fluid is used to determine an upper limit for the time required to establish the steady, exponential distribution of density. It is shown that the steady state will, in in any case in which the initial distribution of particles is uniform, be sensibly attained in that length of time required by one of the particles, moving with the velocity given by Stoke's law, to fall two tube lengths. An examination of Burton's experiment (which sought to prove that the exponential distribution law holds for only very small depths below the surface) indicates that the steady state should have been reached in about 37 days. His failure to find any evidence of an exponential distribution of density after 50 days cannot be ascribed to an inadequacy in the time allowed for settling.

SMALL particles immersed in a fluid in such numbers as to form a dilute suspension experience a steady gravitational drift, the velocity of which is given by Stoke's law of fall, and a Brownian movement due to molecular bombardment. If no other effects such as, for example, electrical or thermal, are present, the two mentioned motions finally produce (whatever be the initial distribution of particles in the fluid) a distribution characterized by the exponential law

$$n = n' e^{\frac{N}{RT} X(x-x')},$$

where $n$ is the number density of particles at a position $x$, the density at $x'$ being $n'$; where $N$ is Avagradro's number, $R$ the gas constant for a gram molecule, $T$ the absolute temperature, and $X$ the gravitational force acting on one of the particles.

It has been suggested by Burton[1] that this exponential distribution is merely a surface phenomenon, in the sense that the above law holds for only very small distances below the surface of the liquid. This contention was based upon the fact that the law had been tested (notably in the well-known work of Perrin) only for small depths, and upon Burton's belief that the law leads to density ratios, when a difference in depth of several centimeters is considered, which are not in accord with physical common sense or experience. Burton therefore experimented with a cell of depth 96 cm, and found, after waiting fifty days for a steady state to be established, no sensible variation of density with depth.

[1] Burton, Proc. Royal Soc. **A. 100**, 414 (1921–22).

Two questions arise in connection with this experiment. Were effects other than the two above postulated present, and was sufficient time allowed for the steady state to be established? No discussion will be made here of the first of these questions, which has been treated in a paper by Porter and Hedges.[2] The present note gives an answer to the second question.

Consider a tube of depth $l$, filled with a liquid of viscosity $\mu$, in which is immersed, at time $t=0$, a uniform distribution of number density $n_0$ of particles of radius $a$ and effective density $\delta$. Set,

$$\alpha = RT/NXl\ ,\quad \beta = 9l\mu/2g\delta a^2\ ,\quad x = yl\ ,\quad \beta t' = t.$$

It is readily checked that $\beta$ is the time required for one of the particles, moving with the velocity given by Stoke's law, to fall a distance $l$; so that $t'=0.5$, for example, corresponds to the time in which a particle would fall half the tube length.

It has been shown[3] that the density $n$ at a depth $x$ at time $t$ is given by

$$\frac{n}{n_0} = \frac{e^{y/\alpha}}{\alpha(e^{1/\alpha}-1)} + 16\alpha^2\pi e^{(2y-t')/4\alpha} \sum_1^\infty \frac{e^{-\alpha m^2\pi^2 t'} m(1 \mp e^{-1/2\alpha})[\sin m\pi y + 2\pi m\alpha \cos m\pi y]}{(1+4\pi^2m^2\alpha^2)^2} \quad (1)$$

where the upper sign is used when $m$ is even, the lower when $m$ is odd. In the paper just referred to, graphs were drawn for the cases $\alpha = 0.025$, 0.1, 0.3, 0.5, 2.0 and for such values of $t'$ as would show the way in which the steady state is set up. This range of values of $\alpha$ was chosen so as to include the value which $\alpha$ had (namely, 0.475) in Perrin's work with gamboge particles. The limited information which these five families of curves afford indicates that as $\alpha$ decreases the time required to set up a steady state increases. This evidence is presented in Fig. 1, drawn from the curves of the previous paper. This figure shows the magnitude of the transient term for the time $t'=0.5$, for the values of $\alpha$ indicated, and for values of $y$ which correspond to the lower half of the tube. It is seen that as $\alpha$ decreases, the transient term becomes larger. Now in the case of Burton's work $\alpha = 1.95\times10^{-6}$, while $\beta = 1.60\times10^{6}$. Insofar as extrapolation from so limited a range to so far removed a value is of any significance whatever, these figures would indicate, together with the above graph, that a very large time might be required for the establishment of the steady state; for the very small value of $\alpha$ indicates a large value of $t'$,

[2] Porter and Hedges, Phil. Mag. **44**, 641 (1922).

[3] Mason and Weaver, Phys. Rev. **23**, 412 (March 1924).

which, in turn, since $\beta$ is large, indicates a still larger value of $t$. The relationship between $t'$ and $t$ is indicated by the fact that the time, fifty days, actually allowed to elapse, corresponds to $t'=2.7$, while the value of $t'=100$ corresponds to over five years.

It is not physically evident whether decreasing $a$ should increase or decrease the time required for the steady state to be sensibly established. Increasing $l$ decreases $a$, and an increased tube length (other factors remaining the same) would naturally lead to an increased time. An increase in $X$, however, also decreases $a$ and an increase in $X$ by, say, increasing the density of the particles would seem to decrease the relative

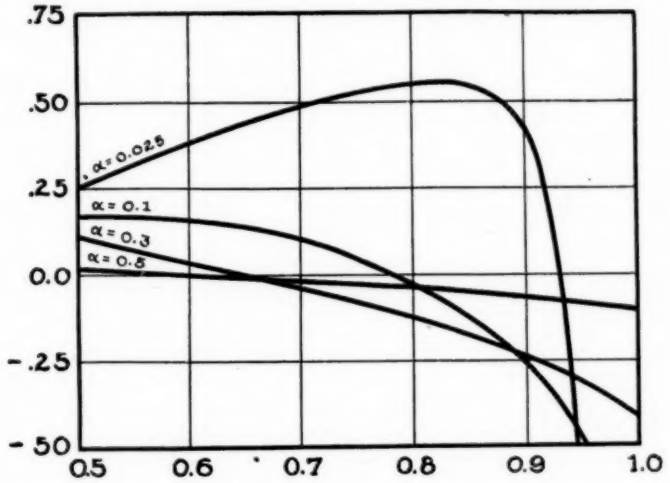


Fig. 1. The magnitude of the transient term at $t'=0.5$. The abscissas range from $y=0.5$ to $y=1.0$, and thus correspond to the lower half of the tube.

effectiveness of the Brownian movement, which tends to maintain a uniform density, as compared to the Stoke's fall.

These considerations lead one to believe it possible that the fifty days allowed were perhaps quite inadequate for the establishment of a steady state. It therefore is desirable to determine this time from the general solution (1). For so small a value of $a$ as characterized Burton's work, the actual calculation of the sum involved is prohibitively laborious, since many hundred terms must be used to obtain the value of $n$ for a single value of $x$ and $t$. An upper limit to the transient term may, however, be easily calculated. In fact, for any value of $y$,

$$\sum_1^\infty \frac{e^{-km^2}(1\mp e^{-1/2a})m \sin m\pi y}{[1+4\pi^2a^2m^2]^2} < (1+e^{-1/2a})\sum_1^\infty me^{-km^2}$$

$$< (1+e^{-1/2a}) \sum_1^\infty e^{-km},$$

the last step following from comparison of the series

$$e^{-k}+e^{-2k}+e^{-3k}+e^{-4k}+e^{-5k}+e^{-6k}+e^{-7k}+e^{-8k}+e^{-9k}+\cdots$$
$$e^{-k}+0\quad+e^{-4k}+e^{-4k}+0\quad+0\quad+e^{-9k}+e^{-9k}+e^{-9k}+\cdots$$

Moreover,

$$\sum_1^\infty e^{-mk}=e^{-k}/(1-e^{-k})\ .$$

Similarly

$$\sum_1^\infty \frac{2\pi a e^{-km^2}(1\mp e^{-1/2a})m^2\cos m\pi y}{[1+4\pi^2a^2m^2]^2}<2\pi a(1+e^{-1/2a})\ \sum m^2e^{-km^2}$$
$$<2\pi a(1+e^{-1/2a})\sum me^{-km}\ ,$$

and

$$\sum_1^\infty me^{-km}=e^{-k}/(1-e^{-k})^2\ .$$

Hence, calling the transient term $H\ (y,t')$, and identifying $k$ with $a\pi^2t'$

$$H(y,t')<16\pi a^2e^{(2y-t')/4a}(1+e^{-1/2a})e^{-k}\left[\frac{1}{1-e^{-k}}+\frac{2\pi a}{(1-e^{-k})^2}\right],$$

or

$$H(y,t')<\frac{16\pi a^2(1+e^{-1/2a})(1+2\pi a-e^{-k})e^{\frac{2y-t'(1+4\pi^2a^2)}{4a}}}{(1-e^{-k})^2}$$

When $a$ is of the order of $10^{-6}$, as in Burton's experiment, and $t'$ is not so large but that $a\pi^2t'$ is also small, this reduces to

$$H(y,t')<\frac{16a}{\pi t'}e^{\frac{2y-t'}{4a}}\ .$$

On account of the size of $a$, the exponential factor decreases with enormous rapidity towards zero as soon as $t'$ exceeds $2y$; and thus, since $y$ never exceeds 1, as soon as $t'$ exceeds 2. For example

| $t'$ | Upper bound for $H$ |
|---|---|
| 2.0000 | $5\times10^{-6}$ |
| 2.0001 | $3\times10^{-11}$ |
| 2.001 | $2\times10^{-89}$ |

Since the curves drawn in the previous article show that as $a$ ranges from 0.1 to 2.0 the steady state is more and more rapidly attained, being

sensibly reached in the slowest instance by $t' = 1.0$; and since the above argument shows that for smaller values of $\alpha$ the steady state is sensibly attained for values of $t'$ just exceeding 2, it is therefore possible to state in general that the steady state will in all cases be reached in that length of time necessary for one of the particles, moving with the velocity given by Stoke's law, to fall two tube lengths.

It is therefore assured that in Burton's experiment, in which $t' = 2.0$ corresponds to approximately 37 days, sufficient time was allowed for the steady state to be established.

DEPARTMENT OF MATHEMATICS,
UNIVERSITY OF WISCONSIN,
January 4, 1926.

# BOOK REVIEWS

**The Theory of Measurements.** LUCIUS TUTTLE and JOHN SATTERLY.—This text, according to the authors, is intended for undergraduate students both in mathematics and in the physical sciences. The treatment is very elementary and a minimum of mathematical knowledge (not even trigonometry) is presupposed. The book, however, includes most of the information on this subject which every graduate student in Physics, and in the other exact sciences, should have but so often unfortunately does not have. It forms an admirable introduction to the more extended accounts of special topics found in more advanced texts. The range of topics is quite remarkable, as can be gathered from a few of the twenty-two chapter headings, taken at random,—Weights and Measures, Significant Figures, Logarithms, the Slide Rule, Graphic Analysis, Interpolation and Extrapolation, Deviation and Dispersion, Least Squares, Applications to Biology. The last portion of the book is designed especially for students in the biological (and medical) sciences. Numerous problems are included with every chapter, and these with many illustrations and typical numerical results, fit it for its avowed purpose as a complete textbook on this subject.—Pp. xii+333, 93 figs. Longmans, Green and Co., New York City, 1925. Price $4.50.

RAYMOND T. BIRGE.

**Helmholtz's Treatise on Physiological Optics. Volume III, The Perceptions of Vision.** Translated from the third German edition. Edited by Prof. James P. C. Southall.

A goodly portion of our knowledge of the world about us may be credited directly to the human organ of vision. That organ is almost immeasurably more powerful than all the other sense organs combined. Its structure and functioning were minutely described and discussed in the first two volumes of Helmholtz's Physiological Optics. The third and final volume deals with the perceptions and impressions obtained through vision.

Now our impression of an observed object is composed of a great many different factors. These are of two classes, what is seen and what is known of this and similar objects; the retinal image, the difference between the two retinal images, muscular and nervous efforts connected with vision and the like constitute the visual function itself but this is but a small part of the story our eyes tell us, namely, our interpretation of that seen. If a stationary object be viewed and the head turned, the object is still seen stationary. If the eyes be turned from side to side it still appears stationary though its image races across the retina. It is only when the whole body or the object viewed is actually moved that motion is sensed. The retinal image is inverted but we see objects erect. If the retinal image is erected (by means of an auxiliary lens) objects are seen inverted but a week or two of continuous use is sufficient to reform the correct interpretation just as with colored glasses or spectacles in even shorter time. Even the defects of the eye, or more properly the departures from strictly orthoscopic imagery, are unconsciously allowed for. Artists, seeking to produce impressions rather than exact reproduction, must exaggerate the barrel distortion, contrasts, chromatic aberration and diffuseness from the center of attention outward in order to produce the impression sought.

Such is the nature of the field which is the subject of this third part of physiological optics. Helmholtz frankly states that the material is but fragmentary but that it is better to make a beginning. This translation is of the third German edition which is practically a reproduction of the first written by Helmholtz in 1866. The numerous

notes and several appendices by von Kries follow closely the original plan. Binocular vision is treated quite fully since the invention of the stereoscope by Wheatstone had just then attracted considerable attention. Whether our concepts of space and time are intuitive or are the result of experience was much debated in those days, Helmholtz holding that our ideas of space are largely empirical and acquired. If criticism be offered such a monumental work it must be first that it is not sufficiently comprehensive in plan. Breadth and balance are lacking although a few subjects are discussed with great thoroughness. Secondly, the treatment is geometrical and philosophical rather than physical and psychological as it properly should be. A proper treatment of the field should be (inter alia) a veritable text book of the fundamental principles of art. Material is now available for filling in nearly all parts of the field attempted and even for an outline of the limitations of visual preception. The translation is excellent, clear and vivid despite the many fine distinctions drawn between concepts but roughly definable. The volume closes with a 16-page bibliography of the recent (1911–1925) literature of the subject and very full indices to both authors and subjects cited. Pp. x+736, 81 figs. Published by the Optical Society of America, and to be obtained from the Secretary, F. K. Richtmyer, Cornell University. $7.

P. G. NUTTING

**Modern Magnetics.** FELIX AUERBACH. Translated by H. C. Booth.—The German edition of this work (*Modern Magnetismus*, Vieweg, Jena, 1921) is a diffuse and uncritical but very readable account of the subject down to about 1915. The arrangement is peculiar, magneto-optics, for an example, preceding electro-magnetic theory. What little mathematics is attempted suffers from the author's bias for graphics and from an unsystematic choice of symbols ($q$ stands for four different things besides being the abbreviation for "curl"). There are a few bad blunders, notably in regard to the equipotential surfaces for a separated pair of unlike poles of equal strength (Figs. 4 and 9, with associated text). The present translation is neither good English nor a fair equivalent for the original German, and contains many absurdities. The specialist can guess from the context that "developed" (p. 199) should be "wound with wire," and that "beam" (p. 133) should be "radiation," but the unwary reader will go astray at least once on every page. The unfitness of the translator may be judged from his invention of the clause (p. 192): "when the permeability of iron becomes small in comparison with that of air." The index is poor. The publisher is to blame for now issuing as "modern" a book which barely deserved that adjective five years ago. The condensation of the table of contents is unfortunate, because the remaining chapter titles are none too descriptive. The cuts contain all the original imperfections (Figs. 65, 74), the half-tones are foggier (Fig. 87) and the new lettering is often slovenly (Figs. 52, 59). A commendable change is the addition of a bibliography, the selection and arrangement being that imposed by the text. The antiquity of most of the 350 references suggests why this book is not conspicuously more modern than the third (1901) edition of Ewing's classic work on a portion of the subject. The paper, printing and binding are adequate. For reasons already stated the book will find little favor with specialists. The American technologist may use it cautiously for numerical data, but can find elsewhere better specifications for magnetic testing than those standard in Germany in 1910. The general reader will derive from it but a confused idea of the present state of knowledge regarding magnetism. Pp. VII+306, 167 diagrams. E. P. Dutton and Co., New York, 1926. $6.00.

L. W. MCKEEHAN

**Popular Research Narratives.** Vol. 2. Collected by the Engineering Foundation.—Here is a group of fifty scientific essays evidently addressed to the lay reader. The language is non-technical; the subject matter very concrete; the discoveries important; the

inventions valuable. It is a collection of wonderful stories—each told within the space of three small pages. The reader is therefore not carried very far; but the growing interdependence of engineering and pure science is made very evident. Much of the pleasure of reading a book is lost when, as in the present case, one does not know who is responsible for the words; the ideas may be ever so good; one wants to know who clothed the ideas with these words. It is unfortunate also that no definite references to literature are given, such as would permit an interested reader to pursue the subject farther. But it is hardly fair to appraise these expositions from a scientific view point, as is usually done in this journal. Most students of physics will be stimulated by the account of *Permalloy*, *The High Temperature Electric Furnace*, *The Sonic Depth Finder*, *Talking Across the Ocean*, *The Geophone*, *Modernizing the Steam Locomotive*, and many others.

The purpose which underlies this little volume has been clearly set forth, in the introduction, by Professor Pupin who says: "Take care of the bee and the honey; enrich the field of human experience. Scientific and engineering research is the honey gathering process; men trained and disciplined in this field of inquiry are the honey gathering bees. Research and the men conducting it contribute to the honey-hearted hive of our national life. We must cultivate both, otherwise the hive will soon be empty." Most men at some period in their lives—generally an early one—have cultivated the honey bee; but the enthusiasm of the youthful apiarist is generally short-lived. The reason for this is that most bees behave as if they were perfectly familiar with Virgil's verses: *Sic vos, non vobis, etc.* At any rate, they insist upon making honey in their own way; simply because it is the greatest fun in the world, the best sport imaginable. Whether the process can be greatly expedited by the men who collect the honey may be fairly doubted. Investigators, if caught young, may be shifted from university laboratories to those of the large corporations, but whether humankind thus derives more benefit than it would from the labors of these same men in perfectly free academic laboratories is another large question. In any event, it is to be hoped this volume will help to remove that illogical distinction between pure and applied science, between what Norman Campbell calls "the roots and branches of the tree of experimental knowledge." viii+174 pp., Williams & Wilkins Company, Baltimore, 1926. $1.00. HENRY CREW

# PROCEEDINGS

OF THE

# AMERICAN PHYSICAL SOCIETY

## MINUTES OF THE MONTREAL MEETING, FEBRUARY 26-27, 1926

## JOINT MEETING WITH THE OPTICAL SOCIETY OF AMERICA

The 137th regular meeting of the American Physical Society was held in the Macdonald Physics Building of McGill University, Montreal, on Friday and Saturday, February 26 and 27, as a joint meeting with the Optical Society of America. The presiding officers were Dayton C. Miller, President of the American Physical Society, and W. E. Forsythe, President of the Optical Society of America. The attendance was about 100 persons.

On Friday morning there was a session of the Optical Society and a session of the Physical Society. The program on Friday afternoon consisted of papers contributed by the American Physical Society, and a lecture by Professor Frank Allen of the University of Manitoba, on "The Second Half of Vision," given by invitation of the Optical Society. The Saturday morning session was a regular session of the American Physical Society.

On Friday evening the members of the American Physical Society and of the Optical Society of America were guests of the Chancellor, the Principal and the Board of Governors of McGill University at a dinner held at the Mount Royal Hotel.

At the regular meeting of the Council held on Friday, February 26, 1926, one new fellow was elected, one member was transferred to fellowship, and fourteen were elected to membership. *Elected to Fellowship:* Abraham Joffé. *Transferred from Membership to Fellowship:* Frank M. Walters, Jr. *Elected to Membership:* John B. Bishop, Donald C. A. Butts, Herbert R. Childs, Frederick W. Cunningham, Frederick W. Doermann, Frederick H. Drake, Peter Fireman, Sydney B. Ingram, Hermann E. Krefft, Horace H. Lagerpusch, Theodore J. Martin, W. C. Phebus, Eugene W. Skinner, and Leo John Vollmayer.

The titles and abstracts of papers presented to the Optical Society of America will be found in the Proceedings of that Society, published in

the *Journal of the Optical Society of America and Review of Scientific Instruments.*

The abstracts of the forty-two papers presented to the American Physical Society are given in the following pages. Papers Nos. 1, 19, 25, 33, 39, 41 and 42 were read by title. Papers Nos. 19, 33 and 35 were received after the program of the meeting had gone to press and were therefore not included in the printed program of the meeting published in Volume 1, Number 7, of the Bulletin of the American Physical Society. An **Author Index** will be found at the end.

HAROLD W. WEBB,
*Secretary*

## ABSTRACTS OF PAPERS

**1. Diffraction by a grating.** G. BREIT, Carnegie Institution of Washington.—Frauenhofer diffraction phenomena have been explained by Duane in terms of transfer of momentum to the diffracting apparatus. It has been shown by Ehrenfest and Epstein using Compton's modification of Duane's theory, as well as by the writer on a different basis, that Duane's explanation may be made quantitative. In the present paper the requirement of proper maintenance of equilibrium in black body radiation is applied. It is shown that Duane's grating does not acquire the proper kinetic energy when exposed to black body radiation if quanta are conceived of as mechanical entities. On the contrary it is necessary to introduce probabilities of interaction similar to those used by Einstein in his 1917 needle quanta paper. In such a way the scattering by a grating is similar to the scattering by free electrons (Pauli) and by Bohr atoms. The conditions for interference come out, therefore, simply as selection rules. This point of view explains also Fresnel diffraction phenomena. This continuous transition from a Bohr atom *via* Hertzian oscillator to a grating is traced.

**2. Investigation of x-ray spectra by the method of visibility.** LACHLAN GILCHRIST, University of Toronto.—Formulæ are derived which represent the distribution of intensity and the visibility in x-ray spectra when the analysing crystal is composed of a large number of parallel reflecting planes. In the development the primary sets of multiple reflections are taken into account and the source is not supposed to be strictly monochromatic, but to have a "width." For a perfect crystal the expression for intensity is $I=[b^2(1-b^2A^6B^4)^2/(1-A^4B^4)][P+2A^2B^2C_1\cos 4\pi Dm+2A^4B^4C_2\cos 8\pi Dm+\cdots\cdots]$ and the expression for visibility is $V=A^2B^2C_1/P$, where $b$ and $a$ are the coefficients of reflection and absorption, respectively. $A=1-a$ and $B=1-b$. $P_1C_1$, $C_2$, etc., $2D$, and $m$ have the significance given to them by Michelson and Rayleigh in articles on interference phenomena. This agrees with the visibility in the spectra obtained with Calcite crystals by Bergen Davis and Stempel. For crystals with random or "irregular" spacing maximum and minimum intensities are given by $I=[b^2\Sigma_p^p A^{4p}B^{4p}/(1-b^4A^4)][P\pm 2b^2A^2C_1]$ and the visibility is given by $V=2b^2A^2C_1/P$. If the intensity at the minimum is zero when perfect crystals are used, which seems to be in accord with experimental results, then in spectra which are obtained by means of imperfect crystals the intensity at the minimum is not always negligible and deductions should not be made from the intensity of the maximum alone.

**3. The fine structure of the K x-ray absorption edge of calcium.** GEO. A. LINDSAY and GEO. D. VAN DYKE, University of Michigan.—The absorption spectrum of calcium was photographed by the use of a Siegbahn vacuum spectrometer. The absorption

occurred in the calcite crystal, thus rendering a screen unnecessary. A gypsum crystal was also used. The edge obtained from the calcite crystal showed two white lines and a wider absorption region farther towards shorter wave-lengths. The gypsum showed one strong narrow white line with faint traces of a second, while the third wider absorption region was about like that from the calcite crystal. The difference between the first and the second white lines was about 13 volts, between the first edge and the beginning of the third, about 30 volts. If the calcium is in the form of an ion in the compound, the K electron which is removed by the absorption is more likely to stop in one of the orbits normally occupied by the valence electrons, or in an orbit somewhat farther out than it would be if the atom were neutral, for the ion has a net charge of +2. This may explain the appearance of the white lines. These multiple edges seem to be found oftener for compounds (which are probably ionized) than for elements.

**4. The scattering of x-rays.** J. A. GRAY, Queen's University.—The x-rays scattered from different thicknesses of material have been examined by an absorption method. The thinner the material, the greater is the average wave-length of the issuing radiation. The results indicate that the modified "band" in scattered x-rays has a greater total intensity than is commonly supposed. From these and other results, it seems possible to explain the small intensity of reflection of x-rays from crystals for the larger angles of scattering. No evidence for the J phenomena has been found.

**5. Cross-sectional area of an Einstein quantum.** R. J. PIERSOL, Westinghouse Research Laboratory.—A gas kinetic treatment of the equilibrium of light quanta and electrons led Burger and Ornstein to the value $a\lambda^2$ for the cross section of a light quantum, where $a$ is a universal, but unknown constant. By calculation from the vibration volume of the electron Marx showed $a=3/4\pi$, or that the area $A=3\lambda^2/4\pi$. It may be proved from the classical Hertzian theory of oscillators that, if the two electric doublets are parallel, the area of thermal absorption $A_t=3\lambda^2/4\pi$, which is identical with the result obtained by Marx from equilibrium considerations. But it is shown, in general, that where there are a large number of radiating and absorbing electrons, oriented at random, the area of electromagnetic absorption is $A_e=\lambda^2/\pi$. Therefore, the unknown constant in the equation of Burger and Ornstein is $a=1/\pi$ instead of $a=3/4\pi$. Numerically this value is very close to that obtained by Marx, and therefore either value satisfies the interference and diffraction of light.

**6. Radiation arising from the formation of helium from hydrogen.** G. E. M. JAUNCEY and A. L. HUGHES, Washington University.—The loss of mass when four protons and two electrons come together to form a helium nucleus signifies a loss of energy which appears as a quantum. Conservation of energy and of momentum must both be satisfied. Four processes are considered: (1) the protons and electrons initially at rest produce a helium nucleus recoiling in the opposite direction of the quantum, (2) the protons and electrons all initially having velocity $\beta c$ produce a helium nucleus at rest, (3) one proton with velocity $\beta c$ meeting three protons and two electrons at rest produces a helium nucleus at rest, (4) one electron with velocity $\beta c$ meeting four protons and one electron at rest produces a helium nucleus at rest. Solving for $\lambda$ and $\beta$ we find: $\lambda=.0004$A in (1), (2), (3) and .0008A in (4); $\beta=.008$ in (1), (2), .03 in (3) and .9995 in (4). The reverse of (4) is the photo-electric effect. By analogy with x-rays, .0004A and .0008A may be considered as critical absorption wave-lengths. This suggests looking for a trace of hydrogen in a tube of helium after the tube has been exposed to the penetrating rays studied by Millikan.

**7. Radiation from the mutual annihilation of protons and electrons.** A. L. HUGHES and G. E. M. JAUNCEY, Washington University.—Mutual annihilation of a proton and an electron on collision giving a quantum of radiation of wave-length $1.3\times10^{-5}$ A has been suggested as the source of the high frequency radiation studied by Millikan.

It can be shown that this simple process cannot simultaneously conserve energy and momentum. Moreover it is difficult to imagine the reverse process, which would require a quantum suddenly, without apparent cause, to disappear, leaving in its place a proton and an electron. Both energy and momentum can be conserved in a *three body* collision. *Two electrons and one proton* colliding result in a recoil electron moving with a minimum velocity .9999995$c$, and a quantum whose wave-length is $2.6\times10^{-5}$A (or less). *Two protons and one electron* colliding result in a recoil proton moving with a minimum velocity .6$c$, and a quantum whose wave-length is $1.9\times10^{-5}$A (or less). The reverse process is as easy to realize as the forward process. An electron (or a proton) colliding with a quantum (having sufficient energy) causes the quantum to disappear leaving two electrons and one proton (or two protons and one electron).

**8. The "reflection" of electrons from the molybdenum target of a Coolidge x-ray tube.** W. R. HAM and MARSH W. WHITE, Pennsylvania State College.—The method previously described by the writers (*Phys. Rev.* **27**, 111, (1925)) was used. This consists essentially of accurately measuring the energy input into the tube and the heat energy received at the anode. An approximate measure of the ratio of the number of impinging electrons to those emitted from the anode is obtained by taking the ratio of $(W_i-W_a)$ to $W_i$, where $W_i$ is the energy input and $W_a$ is the heat energy received by the anode alone. In the present experiments the data have been extended, with improved technique, to include molybdenum. The tube used was a standard molybdenum-button water-cooled type. Several improvements in the experimental arrangements were made, namely: the temperatures of the water used in the flow calorimeter were read to 0.001 degree by Beckmann thermometers, the water supply was changed to an elevated constant temperature bath and arrangements were provided for keeping the room temperature constant. The curve showing the percentage of "reflected" electrons vs. voltage is similar to that for tungsten, rising from 20 to 1.3 kv to a pronounced maximum of 22.5 at 4 kv and rapidly falling off to 3 at 20 kv, where it remains approximately constant up to 30 kv.

**9. Reflection of soft x-rays.** ELIZABETH R. LAIRD, Mount Holyoke College.—This work continues some done in 1922 with a thermoluminescent detector. Electrons from a low temperature cathode fall on a target, the x-radiation produced passes through a tubular slit into a reflector which can be set at different angles or pulled aside. The reflected or incident beam falls on a Schumann plate. For copper at 1000 volts the result indicates total reflection at a glancing angle of about 5°, and very little for larger angles. At 500 volts the transition is less marked. For this voltage strong reflection occurs at 25°, for 265 volts at 35°, and for 160 volts at 8°. For larger angles the reflection at 160 volts is not as strong as at 265 volts, suggesting the passing of a natural frequency. Nickel at 500 and 1000 volts is similar to copper. Aluminum at 265 volts shows little reflection. Computation of critical angles by the Drude-Lorenz formula depends on values ascribed to natural frequencies, and to wave-lengths of maximum energy for a given voltage. Values for copper of 4° and 14° were computed for 1000 and 500 volts. The experiments show that these Schumann plates become less sentisive for equal energy as wave-length diminishes. The experiments will be continued looking towards more precision in measurement. This work was done at the Sloane Laboratory, Yale University.

**10. Electron distribution in sodium chloride.** ARTHUR H. COMPTON, University of Chicago.—Extending Duane's method of Fourier analysis, a series expression is obtained for the mean electron density at different distances from the centers of the atoms in a crystal. Every term in the series may be determined by measurements of the intensities of the reflection of x-rays from the crystal, whereas in Duane's method a constant term remains undetermined. Using the data of Bragg, James and Bosanquet for the reflection

from rock-salt, the electron density is found to fall definitely to zero for the sodium ion 1.1A, and for chlorine at 2.0A. from the center. Sodium has a group of eight electrons (probably K and $L2_2$) near the center with two electrons (probably $L2_1$) at .9A from the center. Near the center of the chlorine ion are 10 electrons (K and L) outside of which are eight others, resolvable into $4(3_3?)$ at .74A, $2(3_2?)$ at 1.14A and $2(3_1?)$ at 1.60A. These distances are uncorrected for thermal agitation. The electron distribution curves are similar to but show more detail than those obtained by Bragg and Havighurst from the same data.

**11. Crystal size and hysteresis in precipitated magnetite.** LARS A. WELO and OSKAR BAUDISCH. The Rockefeller Institute for Medical Research.—Magnetite has been prepared by three methods previously described (*J. Biol. Chem.* **65**, 216, (1925)). These oxides will be referred to as Lefort's, Haber's, and B and W's. In the present study the usual method of preparation of the latter oxide was departed from by boiling in excess of alkali. The oxides are of identical crystal structure but the average crystals differ in size. This is known from the relative sharpness and diffuseness of the diffraction lines. Lefort's oxide has a small average crystal, Haber's an intermediate size, and B and W's the largest crystals. Their magnetic properties have been studied. Barring the unknown but possible influence of impurities (sulphates to the extent of about 2 per cent) unavoidably occluded at the moment of precipitation, the conclusion is that lean hysteresis loops, low coercivities and low remanences are associated with oxides composed of small crystals. The significant constants for Lefort's, Haber's, and B and W's oxides are respectively: permeability maximum at 50, 240 and 380 gausses; coercivity of 14, 67 and 149 gausses; remanence of 8, 18 and 34 per cent. The relative areas of the hysteresis loops are as 1, 5 and 7.

**12. A method for exciting spectra of certain metals.** JOHN K. ROBERTSON, Queen's University.—An examination of the spectrum of tin when excited by the electrodeless discharge has shown that characteristic metal lines are brought out in the case of a discharge in a quartz tube containing residual gas and pieces of cold metal to about the same extent as when the tube was heated to 1000°C. To study the origin of the metallic lines, other metals were examined, in all cases sufficient hydrogen being left in the quartz tubes to carry the discharge. With silver and gold negative results were obtained, but with zinc and cadmium lines were present as in tin. Moreover, with these two metals, only ultimate and penultimate lines of the neutral or the ionized atom appeared. In cadmium, for example, the quartz spectrograph showed with marked intensity only $2144(1\sigma-1\pi_1)$, $2265(1\sigma-1\pi_2)$, $2288(1S-1P)$, $3261(1S-1p_2)$. Evidence is given that the explanation of the effect is found, not in the presence of vapor due to temperature, but as a result of some kind of reaction between active gas atoms and the metal or an oxide of the metal. The appearance of ultimate ionized lines and the non-appearance of higher members of neutral series is, however, somewhat puzzling.

**13. Spectrum of copper.** A. G. SHENSTONE, Princeton University.—In addition to a normal doublet spectrum, the copper arc yields a complicated spectrum built on the low inverted $^2D$ term which connects the two spectra. The Heisenberg-Hund theory predicts low terms $^4P$, $^4D$, $^4F$, $^2P$, $^2D$, $^2F$ which should combine with the $^2D$. Zeeman effects obtained on a Hilger E.1 spectrograph identify the term $\nu=22194.0$ as $^4P_2$ and $\nu=21364.3$ as $^4P_1$. The latter has a $g$ equal to $2.5\pm.1$, (theoretical value $=8/3$). The two terms $\nu=17392.3$ and $\nu=16487.2$ have irregular $g$ values equal to $.21\pm.01$ and $.51\pm.02$. They may be $^4D_1$ and $^2P_1$. The term $\nu=17763.9$ has $g=1.1\pm.1$ which must also be irregular. All the evidence indicates that this portion of the Cu spectrum has the characteristics of a spectrum of the second rank. In all 20 positive terms combining with $^2D$ have been found. These in turn combine with 21 negative terms and account for 175 arc lines including all the important ultra-violet and most of the important

visible lines. Some constant frequency differences have been found connecting the prominent ultra-violet $Cu^+$ lines. Several lines have been identified from Zeeman patterns.

**14. Standard wave-lengths for use in the extreme ultra-violet.** STANLEY SMITH and R. J. LANG, (introduced by R. W. Boyle). University of Alberta.—Wave-lengths of a number of lines, chiefly of carbon, which frequently occur in the spectra of other elements, have been measured in the second and third orders on plates obtained in a vacuum spectrograph with a concave grating of radius 6 feet ruled with 30,000 lines to the inch. The dispersion was approximately 4.5A per mm. The carbon lines and the iron standard lines used as comparisons were obtained *simultaneously* by passing a condensed spark discharge between a carbon and an iron electrode. (1) *Method of measuring.* Details are given of a new method of obtaining the curves to correct (a) for deviation of the spectra from normality (b) for error in the tilt of the photographic plate. These curves can be used for correcting the calculated wave-lengths obtained by interpolation between any pair of standards chosen within the range of the curves. The curves for correction (b) are proved to be very approximately parabolic. (2) *Wave-length measurements* of the following lines have been made: 904, 977, 1010, 1036, 1175, 1215, 1247, 1323, 1329, 1335, 1548, 1550, 1561A.

**15. Stark Patterns.** J. STUART FOSTER, H. B. HACHEY and W. ROWLES, McGill University.—The simplest Stark pattern consisting of one parallel component and no perpendicular component (1/0) is observed for three members of the orthohelium series $2s-ms$. This pattern, together with others previously reported by one of the writers (Nature **116,** 135 (1925)) completes a group of Stark patterns 1/0, 1/1, 2/2, 2/3 which theoretically exist in the Balmer series. The parhelium combination line $2P-6P$, next the diffuse line in the 4144 group, is clearly observed as a separate line in lowest field. The dispersion 3A/mm makes the separation (0.26A) of these lines more definite than in a previous experiment (Phys. Rev. **23,** 667, 1924). The orthohelium lines near and including 4026 are observed as doublets in high electric fields. The observed separation is constant throughout.

**16. A method of measuring relative intensities of Stark-effect components.** J. STUART FOSTER and M. LAURA CHALK, McGill University.—A modified Lo Surdo tube with rectangular cross-section near the cathode is employed. At a given distance from the cathode the light intensity is constant over the central portion of the discharge in the Crookes dark space. The tube is placed with its axis horizontal and with the longer side of the rectangle in a vertical position. Light from very near the cathode is analyzed and found to give straight lines of uniform intensity corresponding to the different Stark-effect components. Using this source and a double neutral wedge, photographs of the $p$ and $s$ components of $H\beta$ are obtained from which the relative intensities are measured. Final results will be published when a check has been made on the wedge calibration.

**17. Interpretation of complex spectra.** OTTO LAPORTE, Research Fellow, Bureau of Standards.—Recent development of the theory of atoms with several valence electrons, permits interpretation of the complex arc and spark spectra of the two long periods of elements. The low and metastable term group of the spectrum of an element with $z$ valence electrons and belonging to the second long period can be accounted for by the three configurations: $(z-2)$ electrons of type $4_3$ and two $5_1$, $(z-1)$ of $4_3$ and one $5_1$, $z$ electrons of $4_3$ and none of type $5_1$. Hund has shown that the first and second kinds represent respectively the normal states of the neutral and simply ionized atoms from Ca to Ni. In the second long period, it is possible to show the presence also of terms belonging to the third configuration in almost every spark spectrum. The tendency to eliminate the $5_1$ electrons gradually becomes stronger towards the end of the period:

Neutral Pd and the singly ionized atoms from Cb to Ag are normally in states with $4_3$ electrons only. The new theory connects term differences in the spark with differences of series limits in the arc spectra. This is illustrated with Cr and Fe.

**18. Approximating the "screening constant."** D. T. WILBER, Cornell University.—A combination is proposed of the Sommerfeldt assignment of azimuthal quantum numbers of 1, 2, 3, 4, to the *s*, *p*, *d*, and *f*, terms, respectively and his value of the term energy, $R(Z-s)^2/n^2$ plus a negligible term, with a proposed approximation value for the "screening constant" $s$. This value takes the form $s=(1-1/(4n)^k)(Z-1)$ and $\nu=R[(1+(Z-1)/(4n)^k)^2/n^2]$ is the energy value of a term of total quantum number $n$ and azimuthal quantum number $k$. Calculating the value of the series terms of the alkalies from this formula gives a good check for the four series of lithium and follows the trend of the sharp terms up to caesium but shows a systematic and finally large deviation in the case of the *p* terms. In general this systematic deviation is proportional to the doublet separation. This formula does not compete with the Rydberg, Ritz or Hicks formulæ for accurate interpolation but does show how Bohr's values of total quantum number can be possible when so much larger than the usual value assigned the terms.

**19. Electronic states of molecules and structure of band spectra.** ROBERT S. MULLIKEN, Harvard University.—Postulates: (1) The electronic states of molecules can be characterized by term designations ($^1S$, $^2p_1$, etc.), each associated with an electronic quantum number $j_e$ substantially identical with *Sommerfeld's* atomic $j$ for the given term-type, and representing the *true* resultant electronic angular momentum of the molecule (or in some cases *each atom* may have its own $j_e$). (2) The vector or vectors $j_e$ set themselves essentially parallel ($\pm\epsilon$ components) or perpendicular ($\sigma$ components) to the vector $m$ representing the nuclear precessional quantum number; the nuclear rotational energy is then given by a Kratzer-Kramers and Pauli formula $F(m)=B(\,(j^2-\sigma^2)^{\frac{1}{2}}\mp\epsilon)^2\,\ldots\,.$; $j$=final resultant of $m$, $\epsilon$, $\sigma$. (3) $j$ has integral values for odd molecules, half-integral values for even molecules, and is subject to the selection principle $\Delta j=0,\pm1$, the transition $\Delta j=0$ (Q branches) being always present except when $\sigma=0$ for both initial and final states, as predicted by the correspondence principle. These postulates, which are the outgrowth of work of Kratzer, Birge, Mecke, and others, appear capable of accounting in a unified way for many of the features of the structure of most known band spectra, and promise important information on molecular structure, orientations and other characteristics of molecular and *atomic* electron orbits, etc.

**20. Critical potentials and the heat of dissociation of hydrogen as determined from its ultra-violet band spectrum.** ENOS E. WITMER, National Research Fellow, Harvard University.—Lyman has found that under certain conditions of excitation the mixture of hydrogen with argon causes the former to emit a very simple, one-dimensional series of bands, lying between λλ 1063 and 1670. These bands are apparently *Q* branches with perturbations near the origin, and, as this investigation indicates, are excited as follows. The energy imparted to a hydrogen molecule is limited to 11.6 volts, the resonance potential of argon. This is just sufficient to raise the hydrogen molecule from the normal state, $e_l=1$, $n=0$ to $e_l=2$, $n=0$, where $e_l$ is the total electronic quantum number, and $n$ the vibrational. Returning, the jumps are $e_l'=2$, $n'=0$, to $e_l''=1$, $n''=0$ to 12. The curve ($n''$, $dE/dn''$) indicates that $dE/dn''$ becomes zero shortly beyond $n''=12$, so that at $n''=12$ the molecule is on the verge of dissociation, and $n''=13$ apparently cannot exist. The data give the following molecular constants: lowest resonance potential=11.61 volts; the heat of dissociation=4.27 volts=98,570 calories in agreement with Isnardi's (Zeit. Elektrochemie **21,** 405, 1915) value of 95,000 calories; the frequency of infinitesimal vibration $\omega_0$ in the normal state=4260 $cm^{-1}$ in fair agreement with the value 4880 $cm^{-1}$, obtained by Kemble and Van Vleck (Phys. Rev. **21,** 653, 1923) from specific heat data. Combining this resonance potential with other spectroscopic data gives 15.2 volts as the probable ionization potential of $H_2$.

**21. Flow of ions through a small orifice in a charged plate.** L. TONKS, H. M. MOTT-SMITH, JR., and I. LANGMUIR, General Electric Co.—To separate the ionic and electronic parts of the electric current to an electrode immersed in an arc, A. F. Dittmer suggested allowing the carriers to pass through small holes in one electrode, B, and collecting them on a second close parallel electrode, C. The undesired carrier can be prevented from reaching C by the potential of either B or C. Practically B must be negative with respect to sheath-edge A, C positive with respect to B. We have calculated the electric field about a circular hole in a thin infinite charged plate, the transverse velocity acquired by a fast charged particle passing through the hole and the effective change in the area of the hole due to focusing action. These results enabled us to calculate the volt-ampere characteristic of C when collecting electrons having a Maxwellian distribution, or ions having Maxwellian and quasi-Maxwellian velocity distributions. The usual semi-log plot of the electron characteristic yields a temperature which is slightly high, the ion characteristic is linear for small accelerating voltages and its semi-log plot for retarding voltages is curved. The determination of current densities and carrier temperatures from the characteristics and the errors involved have been worked out.

**22. Positive ray analysis of the ions in a discharge in hydrogen.** H. D. SMYTH and C. J. BRASEFIELD, Princeton University.—Previous experiments by one of the authors and others indicate the importance of $H_3^+$ as a product of ionization in hydrogen. Considering the possible bearing of this result on the interpretation of the many lines spectrum the authors are now studying the relative concentrations of different types of ion in a striated discharge. The discharge is maintained between a filament and plate 20 cm apart in a tube of 5 cm diameter. In the side of the tube about half way between plate and filament is a third electrode, to which positive ions are drawn. Through a slit in this electrode some of the ions pass into a positive ray analysis apparatus of the type used before. Preliminary results give sharply defined "peaks" which are identified as due to $H_3^+$, $H_2^+$, and $H^+$. When the ions are drawn from the positive column, the $H_3^+$ ions are by far the most numerous though the $H^+$ peak is also strong. The $H_2^+$ peak is very weak increasing only slightly when the pressure is reduced from .2 mm to .03 mm. However at a pressure of .005 mm $H_2^+$ becomes slightly stronger than $H_3^+$. Experiments are being continued.

**23. Analysis of positive ions emitted by a new source.** G. P. HARNWELL, Princeton University, and H. A. BARTON, Harvard University.—Certain iron oxide crystals containing about 1 per cent of some alkali metal or alkaline earth have been developed by the fixed Nitrogen Research Laboratory for use as catalysts in the synthesis of ammonia. It has been found that these substances have also great utility as constant and abundant thermionic emitters of positive ions (Kunsman, *Science*, **62**, 269 (1925)). The writers have analyzed their emission by the use of a mass spectrograph which has been described by Smyth. Separate catalyst samples containing respectively Na, K, Rb, Cs, Mg, Ca, and in one case both Ba and Sr have been shown in this way to emit only the corresponding singly charged positive ion. No doubly charged ions were observed, nor any ions of the other elements composing the catalysts. The alkali metal ions were emitted at lower temperatures than the alkaline earth ions. Particularly in the case of the latter, a preliminary treatment consisting of reduction at red heat in an atmosphere of hydrogen increased the emission.

**24. The passage of charged particles through helium.** A. J. DEMPSTER, University of Chicago.—The penetration of protons with 300 to 900 volt velocity in helium, reported in a previous paper, has been found to hold for much slower rays. With 14 volt rays the charged bundle of rays was observed at a helium pressure that would give six collisions for the path completed. The average loss of velocity per collision for the

majority of the rays of 900 volt speed in penetrating 15 to 25 helium atoms is less than half a volt per collision. Other ions are neutralized or dissociated in helium, and an approximate free path may be deduced from the rate at which they disappear as the pressure is increased. With hydrogen molecule ions the free path for dissociation of 900 volt rays is about 17 times the kinetic theory mean free path for helium atoms. With slower rays (340 volts) the free path is still larger, 2.5 times greater than for the faster rays. Singly charged helium atoms are rapidly neutralized. If we calculate the number of collisions assuming an ion diameter equal to the atomic diameter given by the kinetic theory of gases, the helium ion makes, on the average, only 2.8 collisions before neutralization.

**25. Report on the failure of the mercury to gold transmutation experiment.** H. HORTON SHELDON and ROGER S. ESTEY, New York University.—During the past year we have repeated the mercury to gold transmutation experiments of Dr. Miethe of Berlin and we have not been able to produce the slightest trace of transmuted gold. Our runs were made in three lamps; one horizontal, one vertical, and one a copy of that used by Miethe. The first ran for 30 hours, the second for 50 hours, and the third for 172 hours. The mercury from each run was carefully analyzed and the analyses checked with blank tests of minute known amounts of gold. We found no transmuted gold whatever, although a quantity fifty times smaller than Miethe claims to have produced in 197 hours was detectable with certainty. It is claimed that transmutation is due to instability of the arc conditions. Our experiments seemed to show more instability than that reported by Miethe, thus refuting that suggestion. The suggested explanation of a change of the number of electrons in the nucleus changing mercury to gold seems good in theory, but incredible in fact, for the potential drop per mean free path of a mercury molecule is only about 0.1 volt in these arcs.

**26. A method of high precision for the determination of critical potentials and the ionization potential of mercury vapor.** ERNEST O. LAWRENCE, National Research Fellow, Yale University.—Electrons from a hot cathode are bent into circles by the magnetic field of 60 cm Helmholtz coils, a portion thereof passing through a series of slits on a circle of 5 cm radius. Using a positive ion sheath to neutralize space charge, currents of the order of $10^{-9}$ amp. for ten-volt electrons emerge from the final slit. Retarding potential measurements show that the electrons have a sharp upper limit to their velocities which is the condition necessary for sharp critical potential curve "breaks" and ultimate precision in critical potential measurements. Such a beam of electrons was passed into the effectively field free space of a Faraday cylinder containing mercury vapor and a suitable electrode arrangement for detecting the formation of positive ions. Observing the potential of the cylinder at which ionization is detectable and the potential requisite to prevent the entrance of electrons gave the value 10.4 volts as the ionization potential of mercury vapor with a possible error of about one-half of one per cent. The method is so precise that the errors no longer lay in corrections for the curve breaks but in the measuring instruments.

**27. Ionization potential and radius of the atom.** A. S. EVE, McGill University.—The ionization potential is roughly inversely proportional to the radius of the atom as already shown for Group I and II in the periodic table (Nature **107**, 552 (1921)). This relation holds well for the rare gases whether the kinetic theory or Bragg's x-ray radii are selected. Glockler (*Phil. Mag.*, **50**, 997 (1925)) has directed attention to this relation, which is investigated more closely in the present communication.

**28. Photo-ionization of a gas by a discharge in the same gas.** F. L. MOHLER, Bureau of Standards.—Measurements were made in a tube containing two thermionic units electrically screened from each other; one serving to give a discharge at measured current and voltage, the other operated at a constant low voltage so that the electron

emission was limited by space charge. Photo-ionization of the gas in the second unit was detected by the increased thermionic current, resulting from neutralization of the space charge. Relatively little photo-ionization is produced by the arc spectrum while a strong effect is produced when the discharge voltage is sufficient to produce higher stages of excitation. The method is therefore well adapted for measuring higher critical potentials. Some of the results obtained are as follows: Caesium; ionization of $6_2$ shell 13.0 volts, double ionization 21.5, ionization of $6_1$, 39.0—Potassium; double ionization 32, ionization of $3_1$ shell 48—Argon; double ionization 34.8, ionization of $3_1$ shell 39.6—Neon; double ionization 54.9. In each case the most pronounced increase in photo-ionization came at the double ionization point and spark excitation stages at slightly lower voltages were also found.

**29. Reconciliation of experiments on probability of ionization by electron impact.** K. T. COMPTON and C. C. VAN VOORHIS, Princeton University.—Recent work in this field by Hughes and Klein, Jesse and the authors, shows considerable lack of agreement. It is shown that that of Hughes and Klein and of the authors is brought into fair agreement by correcting for three sources of error: (1) effect of the electric field on the effective area of holes in the grid through which primary electrons pass; (2) presence of slow secondary electrons and lack of homogeneity in velocities of primary electrons; (3) warming of gas by filament. These three corrections were determined for an apparatus built like Hughes and Klein's. The second one only applies to our earlier results. As a result of such corrections our earlier values should be increased by about 10 per cent at electron velocities above 100 volts and somewhat more at lower velocities, and Hughes and Klein's results and ours are brought into approximate agreement. We could not reconcile Jesse's results with the others. Final corrected values of probability are given.

**30. Small Einthoven galvanometer.** HORATIO B. WILLIAMS, Columbia University.—Theoretical work undertaken for another purpose pointed to the possibility of designing a very small string galvanometer. Criticism of the theory and comparison of its consequences with experimental data having shown its probable soundness, the instrument was designed and built. Its performance corresponds with predetermination. It weighs 5.2 kg which may be compared with 100 kg, the weight of the original instrument, and 18 kg, the weight of the smallest accurate and sensitive string galvanometer hitherto available. The stray field has been much reduced, and at a short distance from the instrument is of the same order as that from an ordinary ammeter with permanent magnets at the same distance. The powerful optical system of the larger instruments is retained. Strings are mounted in interchangeable carriers. The instrument is robust and portable and at ordinary sensitivities very little affected by mechanical vibration. A sensitivity of $5\times10^{-10}$ ampere/cm has been easily attained and is probably not the limit of its performance.

**31. A precision determination of the electrical conductivity of concentrated aqueous solutions of calcium chloride.** MARGUERITE CROW, McGill University. (Introduced by A. Norman Shaw).—Measurements are reported which give the electrical conductivity of solutions of calcium chloride for concentrations of from 25 percent to 50 percent and for temperatures from 10°C to 30°C with an increased precision of 5 parts in 10,000. The following are sample readings near the upper and lower limits of that part of the range over which accurate measurements are new:

| Temperature | Concentration by weight | Specific conductivity |
|---|---|---|
| 16.50°C | 46% | .05482 |
| 11.02 | 38.35 | .09314 |
| 26.10 | 46 | .07349 |
| 26.43 | 38.35 | .13695 |

The determinations of temperature coefficients are given for a more extended range than heretofore; for example, at 18°C, the temperature coefficient at 45% is .0320, whereas at 20%, it is .0197. Variations of this coefficient are tabulated. The method consisted of an application of the standard Washburn method for precision conductivity measurements including auxiliary calibrations and the development of a simple technique for obtaining densities of deliquescent solutions and for transferring them to the conductivity cell, without the opportunity for evaporation. These data were required in connection with the determination of vapor pressure by the measurement of electrical conductivity of thin films of solutions as used in an electrical hygrometer; but the results appear to be of independent value for extending existing tables of conductivity.

**32. The redetermination of the temperature coefficients and the ageing corrections for new and old standard cells of the normal and the acid types.** H. E. REILLEY and A. NORMAN SHAW, McGill University.—The use of "ageing" corrections has made it possible to determine the relative "international mean" for the electromotive force of a normal standard Weston cell from the measurements of a few old cells with a degree of accuracy equal to that obtained by using large numbers of new cells, actually within six microvolts. This has been checked by ten formal exchanges of cells with the Bureau of Standards and the National Physical Laboratory, Teddington, over a period of seventeen years, giving close agreements of high precision. The following sample figures indicate the variation in the ageing for a particular batch of five normal cells, of which the average deviation from their own mean did not vary more than three microvolts in seventeen years: *Ageing correction in microvolts for normal cells made in 1908*; 1909, +6; 1912, +28; 1919, +54; 1926, +68. The temperature coefficients of old normal cells remain unchanged, while those of acid cells more than two years old appear to be relatively erratic. Rules for procedure in analyzing the data of cell comparisons have been reformulated.

**33. Relativity and radiation.** A. S. EVE, McGill University.—Equate the relativity expression for mass to the corresponding quantum value, and we have for the *complete* disappearance of mass $m$, $h\nu = mc^2$. But $\nu = c/\lambda$, so that $\lambda = h/mc$ (Home). Now give $m$ (1) the mass of an electron, (2) the mass of a hydrogen atom, (3) the change of mass when 4 H atoms form one He atom; viz., $m = 1/1800$, or $= 1$, or $= .032$ times $1.66 \times 10^{-24}$. Then the corresponding values of $\lambda$ are $2.3 \times 10^{-10}$, $1.3 \times 10^{-13}$ (Jeans), $4 \times 10^{-11}$ (Millikan); and of $T$ are $4 \times 10^{19}$, $7 \times 10^{12}$ (Jeans), $2 \times 10^{11}$ degrees absolute, if we put $h\nu = mc^2 = (3/2)\, k\, T$. Dr. King prefers $\lambda_{max} T = c = 1.447$, so that if $\lambda_{max} = 1.3 \times 10^{-13}$, $T = 11 \times 10^{12}$, not far from the $7.5 \times 10^{12}$ which Jeans suggests.

**34. The Hall effect in sodium and potassium.** E. S. BIELER, McGill University.—It is not generally recognized that there is one group of metals for which the Hall effect is strictly normal, as far as present measurements indicate, namely the metals of Group I of the periodic table. Data for Cu, Ag, and Au are fairly numerous and concordant; those for the alkali metals are scanty. Tests have been carried out on samples of Na and K filtered in a vacuum when in the molten state, and cast in glass cells of suitable shape. The results indicate that the Hall effect is what would be expected if these metals have one conducting electron per atom. The experiments are being carried on with Li, and will be continued with Rb and Cs.

**35. Thomas Young's Eriometer.** W. F. EMMONS, McGill University, (introduced by A. S. Eve).—Adaptations of the eriometer have been evolved in two forms: (1) A large scale laboratory apparatus for projection or exact measurement. (2) A portable form with dry cells and electric bulb, giving measurements of the diameters of the spores of fungi or of blood corpuscles. The coronas of blood smears on glass slides permit diagnosis of pernicious anaemia owing to a distinct increase of diameter in such cases. The portable instrument can be calibrated by the standard laboratory instrument.

**36. The electro-magnetic equations of light propagation in molecular media of varying density.** LOUIS V. KING, McGill University.—If $(X, Y, Z)$ be the electric intensity in the incident light-wave at $(x, y, z)$, it is customary to add to the displacement-current $(1/4\pi)\partial X/\partial t$ the convection current $ne\,dx/dt$ arising from the motions of the dispersion charge $e$ at $(x, y, z)$ having space-density $n$. The equation (1) $(u, v, w)=(1/4\pi)\partial(X, Y, Z)/\partial t+ne(\dot{x}, \dot{y}, \dot{z})$ for total current densities $(u, v, w)$ at $(x, y, z)$ is usually stated to follow from the *fundamental* Maxwell-Lorentz equations (2) $(u, v, w)=(1/4\pi)\partial(X, Y, Z)/\partial t+\rho(\dot{x}, \dot{y}, \dot{z})$ $\rho$ being, however, the space-density of electricity supposed continuously (and uniformly) distributed. In fact, a closer examination requires us to replace (1) by three equations of the type (3) $u=(1/4\pi)\partial X/\partial t+ne\dot{x}+(1/4\pi)\int[(\xi-x)/r^3]\left[\partial(ne\dot{\xi})/\partial\xi\right]dV$, where $r^2=(\xi-x)^2+(\eta-y)^2+(\zeta-z)^2$, the volume integral being taken to include all dispersion charges. It thus appears that (1) as derived from (2) is exact only for media of constant density and of unlimited extent. The additional term in (3) may be expected to give rise to second order phenomena such as *light scattering* arising from elastic density fluctuation due to thermal agitations.

**37. Cavitation in the track of an ultrasonic beam.** R. W. BOYLE and G. B. TAYLOR, University of Alberta.—In a short paper in 1922 the first of the authors pointed out the possibility of the amount of sound wave energy transmissible through a fluid being limited by a phenomenon of cavitation, the evidence of which would be the formation of bubbles in the track of the waves. This phenomenon has been observed experimentally in the case of ultrasonic waves, where the energy is concentrated in the form of a narrow angle beam. Considering only hydrostatic pressure $(p_0)$ as a preventive of disruption in the medium, the maximum energy transmissible across a square centimeter would be $p_0^2/2\rho c$, $\rho$ being the density of the medium and $c$ the velocity of the waves in it. But experiments, in a preliminary study of this phenomenon, have shown that bubbling is more readily produced in highly volatile liquids; in which case, taking vapor pressure $(p_v)$ into account, the maximum energy transmissible per sq. cm would be $(p_0-p_v)^2/2\rho c$. The preliminary experiments show that cavitation occurs at much lower energies still, the energies being measured by the method of the torsion pendulum. It is evident that this phenomenon can limit the amount of energy transmissible in sound waves.

**38. The relation between the thickness of a partition in a medium and its reflection of sound waves—by the ultrasonic method.** R. W. BOYLE and J. F. LEHMANN, University of Alberta.—Experiments carried out by the authors a few years ago by the method of ultrasonic waves—which offer convenience in this type of work—throw light on the problem of the reflection of sound waves by serving to confirm fairly well the existing mathematical theory by Rayleigh. From this theory, which in one aspect is approximate only, it can be deduced that at perpendicular incidence there should be maximum reflection of sound energy when the thickness of the partition is one-quarter the length of the wave in its material, and there should be no reflection but perfect transmission (neglecting absorption) when the thickness is a half wave-length. In the use of a torsion pendulum to make energy measurements in ultrasonics, a means is offered to test these points experimentally. Such tests gave the best thickness of vane to be used for torsion pendulums in absolute measurements. Incidentally from the results of these tests the modulus of elasticity and the velocity of sound in the material of the pendulum can be determined.

**39. Measurements of street noise in New York City.** E. E. FREE, 175 Fifth Ave., New York City.—A systematic survey has been made of the street noise of New York City, using the 3-A audiometer devised by the Bell Telephone Laboratories, Inc. A standard buzzer note produced by this instrument is allowed to enter the ear of the observer, simultaneously with the street noise. The intensity of the buzzer note is adjusted until it just masks the street noise. Street noise was found to be due practically

entirely to traffic, the automobile truck being by far the greatest noise producer, with street cars and the Elevated Railway making much smaller contributions to the noise level. The physical intensity of the street noise, estimated by calibrating the ear of the observer, was found to vary between approximately .001 dyne per square centimeter and approximately 20 dynes per square centimeter.

**40. The production and measurement of a beam of neutral molecules.** THOMAS H. JOHNSON, Yale University.—A beam of mercury vapor was formed by a condensation method similar to that used by R. W. Wood (Phil. Mag. 30, 300 (1915). Mercury was heated to a vapor pressure of 8 cms. The vapor passed from the heating vessel through two holes 0.25 mm in diameter which were separated by a liquid air cooled chamber 8 cms long, thus permitting a sharp beam of vapor to issue from the second hole into an evacuated region beyond. An ionization gauge of the Dushman and Found type having a capacity of 20 cm was used to explore the beam. The guage was completely closed except for a slit 0.05 mm by 0.2 mm and was so mounted that the slit could be moved along a line intersecting the beam at a right angle at a distance of 8 cms above the last hole. When the slit was placed in the beam the pressure in the gauge increased from $1\times10^{-6}$ to $3\times10^{-6}$ mm. The half-value period for pressure changes was about 3 secs. An experiment is now in progress in which the beam is to be sent through a non-uniform magnetic field and the magnetic moment of the mercury atom measured by the deflections observed with the ionization gauge.

**41. Thermal expansion of tungsten.** PETER HIDNERT and W. T. SWEENEY, Bureau of Standards.—This paper gives the results of an investigation on the linear thermal expansion of tungsten (99.98 per cent) over various temperature ranges between −100 and +500°C. The expansion of tungsten is given by the following empirical equation: $L_t=L_0[1+(4.28t+0.00058t^2)10^{-6}]$, where $L_t$ represents the length of the metal at any temperature $t$ between −105 and +502°C, and $L_0$ the length at 0°C. Average coefficients of expansion for various temperature ranges are given in the following table.

*Average Coefficients of Expansion per Degree Centigrade.*

| | | | |
|---|---|---|---|
| −100 to −50°C | $4.2\times10^{-6}$ | −100 to 0°C | $4.2\times10^{-6}$ |
| − 50 to 0 | 4.3; | 0 to 100 | 4.3; |
| 0 to +50 | 4.3; | 0 to 200 | 4.4; |
| + 50 to 100 | 4.4; | 0 to 300 | 4.5; |
| 100 to 200 | 4.5; | 0 to 400 | 4.5; |
| 200 to 300 | 4.6; | 0 to 500 | 4.6; |
| 300 to 400 | 4.7; | −100 to 500 | 4.5. |
| 400 to 500 | 4.8; | | |

**42. Heat transfer in the annular space between two coaxial cylinders.** S. R. PARSONS, University of Arkansas.—Two brass tubes are mounted with a common axis, one inside of the other, and are warmed by driving hot air, in turbulent flow, through the annular space between them. The rates of transfer of heat to the two tubes are compared by means of a coefficient representing heat transmitted per unit time per unit area of metal surface per unit difference of temperature between metal and air. Lees has shown that in streamline flow there would be a difference in the surface friction per unit area for the two tubes, and if that were true of turbulent flow, there should be a corresponding difference in the coefficients of heat transfer. Experiments show that if there is such a difference, it is not of the magnitude that would be expected from Lees' equations, and fail to show with certainty that any difference exists.

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